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Discrete-time hybrid control in Borel spaces: Average cost optimality criterion

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ABSTRACT

This paper addresses an optimal hybrid control problem in discrete-time with Borel state and action spaces. By hybrid we mean that the evolution of the state of the system may undergo deep changes according to structural modifications of the dynamic. Such modifications occur either by the position of the state or by means of the controller's actions. The optimality criterion is of a long-run ratio-average (or ratio-ergodic) type. We provide the existence of optimal average policies for this hybrid control problem by analyzing an associated dynamic programming equation. We also show that this problem can be translated into a standard (or non-hybrid) optimal control problem with cost constraints. Besides, we show that our model includes some special and important families of control problems, such as those with an impulsive or switching mode. Finally, to illustrate our results, we provide an example on a pollution-accumulation problem.

1. Introduction

Hybrid systems have become a common tool for the analysis and control of complex systems where both continuous components (analogue) and discrete components (digital) interact and the term "hybrid system" is used to represent a variety of cases covering many situations (see, for instance, Branicky [10], Goebel et al. [12] or Lygeros [19]).

In the hybrid system under consideration here, the evolution is given by a standard (or usual-type) sub-dynamic running under almost any situation, but, due to special events (with internal or exogenous causes), the first sub-dynamic is no longer valid and a special sub-dynamic (so-called impulse-type or event-driven-

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type) becomes active, overruling the standard evolution. Two usual-type controls are considered, one for each sub-dynamic, but there is another control which determines which sub-dynamic is active. This control is permitted only when the state of the system is located in some subset of the state space. Moreover, the state itself is a pair where the first component describes the standard evolution of the system (or fast-type variable) and the second component (or slow-type variable) records the structural changes, namely the activations of the special sub-dynamic. In this context, we consider the problem of finding a policy, containing a mix of standard actions and special actions, in order to minimize an infinite horizon average cost.

Hybrid control systems have been extensively studied for continuous time models; see for instance, Bensoussan and Menaldi [6,7], Borkar et al. [9], Branicky et al. [11]. The discrete time case appears in many fewer references: Abate et al. [1,2], Summers and Lygeros [27]. We note that many works are devoted to particular cases of hybrid control, e.g., impulsive control problems (Bensoussan [3], Bensoussan and Lions [4,5], Menaldi [20], Robin [23,24], Stettner [25,26]), switching control problems (Bensoussan and Lions [5], Blankenship and Menaldi [8], Zhang et al. [31]), and standard control problems (Bensoussan [3], Hernández-Lerma [13], Hernández-Lerma and Lasserre [14,15], Puterman [22], and the references therein). Several of these works include the case of the average cost criterion. It is also worth mentioning the references Tkachev and Abate [28] and Tkachev et al. [29], which provide insight on some interesting problems that could be studied under the perspective of hybrid control, as dealing with non-additive criteria involving reachability of subsets of the state space, or verification of specifications both on finite or infinite horizon problems.

It may be convenient to recall that, in a hybrid model in discrete time, there are transitions which do not result in an increase of a "unit of time" and, therefore, one of the key differences between conventional and hybrid models lies on the objective function, and not on the dynamic system itself (which may not be the case in a continuous-time situation). In the paper Jasso-Fuentes et al. [16], the problem of minimizing an infinite horizon discounted cost was addressed with a state dependent discount factor. For the average cost case, we consider an auxiliar ergodic dynamic programming equation (DPE) which is studied by using results on Markov decision processes (see Hernández-Lerma [13] and Hernández-Lerma and Lasserre [14]).

Our main results are the existence of an optimal average policy for the hybrid control problem, the characterization of the optimal average cost as a solution of the DPE, and the equivalence of our problem to a non-hybrid control problem with a cost constraint.

The structure of the paper is as follows: Section 2 introduces the dynamics of the system, the control policies we are going to deal with, and some preliminary assumptions. In Section 3 we define the ratio-average optimality criterion and the DPE, and we establish the existence and characterization of average optimal control policies by means of this DPE. A useful characterization that signalizes the accurate times between changes of sub-dynamics in terms of the so-named contact set is also provided. In Section 4, we present an illustrative example on pollution accumulation. Finally, Section 5 is devoted to analyze an alternative formulation as a non-hybrid control problem which takes the form of a linear programming problem over a set of measures and, to conclude, Section 6 provides a comparison of our hybrid model with impulse and switching control problems.

Notation and terminology

- We recall that a Borel space is a measurable subset of a complete and separable metric space.
- Any metric space Z will be endowed with its Borel σ-algebra $\mathcal{B}(Z)$ and measurability (of sets and functions) will be always referred to the corresponding Borel σ-algebras.
- Given some metric space Z, the family measurable bounded functions on Z (that is, with $||u|| = \sup_{x\in Z} |u(z)| < \infty$) will be denoted by $\mathbb{B}(Z)$. If, in addition, u is nonnegative, we will write $u \in \mathbb{B}^+(X)$.