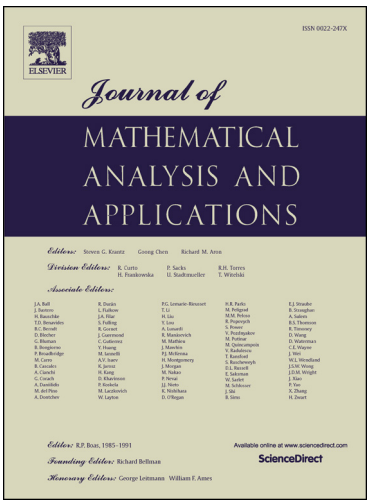
Accepted Manuscript



Conditioning of copulas: Transformations, invariance and measures of concordance

Sebastian Fuchs

PII: S0022-247X(18)30131-8
DOI: https://doi.org/10.1016/j.jmaa.2018.02.014
Reference: YJMAA 22026

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 6 September 2017

Please cite this article in press as: S. Fuchs, Conditioning of copulas: Transformations, invariance and measures of concordance, *J. Math. Anal. Appl.* (2018), https://doi.org/10.1016/j.jmaa.2018.02.014

Conditioning of copulas: Transformations, invariance and measures of concordance

Sebastian Fuchs

Faculty of Economics and Management, Free University of Bozen–Bolzano, Piazza Università, 39100 Bozen, Italy

Abstract

In the present paper we study the problem of how to transform a copula for an arbitrary distribution function into a copula for its conditional distribution function where conditioning is meant with respect to a tail event in which the observations lie below some threshold. To this end, we consider conditioning of copulas as a map which transforms every copula into another one. Besides the general case, which refers to conditioning in all coordinates, we also pay attention to the special case of univariate conditioning, which refers to conditioning in a single coordinate. We investigate the behaviour of conditioning under composition and with respect to certain transformations of copulas, and we show that invariance of a copula under conditioning is equivalent to invariance of a copula under univariate conditioning in each coordinate. Finally, we apply conditioning of copulas to Sklar's Theorem and to measures of concordance.

Keywords: Copula, Conditioning, Invariance, Measures of concordance, Sklar's Theorem, Transformations

1. Introduction

In the present paper we study the problem of how to transform a copula for an arbitrary distribution function into a copula for its conditional distribution function where conditioning is meant with respect to a tail event in which the observations lie below some threshold, and hence with respect to a Borel set with positive measure. Besides the general case, which refers to conditioning in all coordinates, we also pay attention to the special case of univariate conditioning, which refers to conditioning in a single coordinate. In contrast to the literature, we do not assume that the coordinates of the distribution function are continuous, such that the initial copula and the transformed copula may fail to be unique.

Conditioning (or *truncation*) of copulas (also called *conditional copulas*, *threshold copulas* or *tail dependence copulas*) is a well–studied object. This is mainly due to its potential in describing and studying conditional dependence between random variables under a tail event, but also due to various applications in finance and reliability theory: This includes [5, 23, 24] for studying tail dependence, [8] for modeling market contagion, [1, 3, 22, 25] for the construction of systemic risk measures like *conditional value at risk* (CoVaR), [4, 7] for modeling credit derivatives and stock returns and [12, 30] for modeling bivariate ageing. In this context, copulas which are invariant under conditioning are of particular interest (see, e.g., [2, 4, 9, 10, 18, 29, 31]) since for such copulas the values of copula–based measures of association (like measures of concordance) remain unchanged under conditioning. For a comprehensive overview of univariate conditioning including theoretical results but also further applications, we refer to [19].

Here we consider conditioning as a map which transforms every copula into another one, and we investigate its behaviour under composition but also with respect to permutations and reflections. In addition, we show that invariance of a copula under conditioning is equivalent to invariance of a copula under univariate conditioning in each coordinate. In the bivariate case this result was proved in [9] using the fact that the copulas which are invariant under conditioning belong to the

*Corresponding author
Email address: sebastian.fuchs@unibz.it (Sebastian Fuchs)