J. Math. Anal. Appl. 462 (2018) 577–589

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Uniqueness and global stability of positive stationary solution for a predator–prey system

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ARTICLE INFO

Article history:
Received 11 September 2017
Available online 13 February 2018
Submitted by Y. Du

Keywords:
Predator–prey model
Positive stationary solution
Spatial heterogeneity
Global stability

ABSTRACT

In this paper, we are concerned about a predator–prey model with Beddington–DeAngelis functional response, in which a protection zone is set for the prey. In a previous paper, it has been shown that the coefficient k measuring the mutual interference between predators in the Beddington–DeAngelis functional response plays a crucial role in the predator–prey system. As k is large enough, there exists a unique globally stable positive stationary solution except a special case. By introducing an equivalent system, this paper reveals that the uniqueness and global stability still holds true in the special case.

1. Introduction

Since the predator–prey system can generate complex spatiotemporal patterns, it has been extensively studied in the past few decades. It is known that the functional response plays a crucial role in the predator–prey system, and no single functional response can describe all interactions between the species. In particular, the experimental observation implies that the mutual interference among predators may decrease the feeding rate [10]. In 1975, Beddington [1] and DeAngelis et al. [4] introduced the Beddington–DeAngelis functional response

$$\frac{u}{1 + mu + kv},$$

where k measures the mutual interference between predators. Moreover, statistical evidence shows that the Beddington–DeAngelis functional response can better describe the predator feeding over a range of

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https://doi.org/10.1016/j.jmaa.2018.02.032

predator–prey abundances [16]. For the work of the Beddington–DeAngelis functional response, one can refer to [2,5,9,17,19] and references therein.

For the predator–prey model, it is profoundly affected by the spatial environment. In most predator–prey interactions, the prey species would extinguish if the growth rate of the predator is too large or the predation rate is too high. To save the endangered prey, human often sets protection zone for the prey. For the study of protection zones, one can refer to [3,6–8,11] and references therein. In [8], Du and Shi investigate a predator–prey system with Holling-II functional response and a protection zone for the prey. The result reveals that the protection zone produces a critical value, which is independent of the functional response. When the protection zone is over and below the critical patch size, essentially different result can be observed. Then we are interested in that whether the functional response influences the critical value? Can the combination of the protection zone and the Beddington–DeAngelis functional response yield new results?

Following this line of thinking, Wang and Li [18] considered a predator–prey system with Beddington–DeAngelis functional response and a protection zone for the prey species:

$$\begin{cases} u_t - \Delta u = u\left(\lambda - u - \dfrac{a(x)v}{1+mu+kv}\right), \quad x \in \Omega, \qquad t > 0, \\ v_t - \Delta v = v\left(\mu - v + \dfrac{cu}{1+mu+kv}\right), \quad x \in \Omega \setminus \overline{\Omega}_0, t > 0, \\ \partial_\nu u = 0, \quad x \in \partial\Omega, t > 0, \quad \partial_\nu v = 0, \quad x \in \partial\Omega \cup \partial\Omega_0, t > 0, \\ u(x,0) = u_0(x) \geq 0, \quad x \in \Omega, \quad v(x,0) = v_0(x) \geq 0, \quad x \in \Omega \setminus \overline{\Omega}_0, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^N (N \geq 1)$ with smooth boundary $\partial\Omega$, Ω_0 is an open and connected subdomain of Ω with smooth boundary $\partial\Omega_0$. $u(x,t)$ and $v(x,t)$ represent the population densities of the prey and predator species, respectively; λ, c, m, k are positive constants and μ is a real constant, which may take negative values; the function $a(x)$ satisfies $a(x) \equiv 0$ in $\overline{\Omega}_0$ and $a(x) = a > 0$ in $\overline{\Omega} \setminus \overline{\Omega}_0$. Here, Ω_0 is the protection zone for the prey, in which there are no predator and no predation.

The result in [18] reveals that rather different from the Holling-II functional response, the Beddington–DeAngelis functional response has an important impact on the critical value except the protection zone and it diminishes the critical value obtained in [8]. Comparing to the Holling-II functional response, the Beddington–DeAngelis functional response has an extra term kv. Thus, one sees that the coefficient k plays an important role. Then the effect of large k is further studied. As k is large enough, we show that (1.1) has a unique positive stationary solution. Moreover, it is globally stable.

Unfortunately, it should be pointed out that due to the technical reason, the uniqueness and global stability cannot be given in the case of $\mu = 0$. First, the theory of the fixed point index is used to study the multiplicity of the positive stationary solution, whereas we cannot calculate the degree of certain nonnegative stationary solution in the case of $\mu = 0$. To address this problem, we adopt the idea used by Peng and Shi [13], which is introduced by [14,15]. Through an equivalent stationary problem, the implicit function theorem works well to show the uniqueness. Precisely, we have the following result:

Theorem 1.1. *There exists a positive number K_0 such that for $k \geq K_0$, (1.1) with $\mu = 0$ has a unique positive stationary solution $(u_k(x), v_k(x))$.*

Second, since $\mu = 0$ and $v_k(x) \to 0$ as $k \to \infty$, we cannot estimate the decay rate of $|v(x,t) - v_k(x)|$. To overcome this difficulty, we introduce the same equivalent system. By some more precise estimations, the global stability of the unique positive stationary solution can be shown. Precisely, we have the following result: