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Anticipating stochastic equation of two-dimensional second grade fluids

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ABSTRACT

In this paper, we consider a stochastic model of incompressible second grade fluids on a bounded domain of $\mathbb{R}^2$ driven by linear multiplicative Brownian noise with anticipating initial conditions. The existence and uniqueness of the solutions are established.

1. Introduction

In this article, we investigate the existence and uniqueness of solutions of the following anticipating stochastic equation of second grade fluids:

$$\begin{cases} d(u-\alpha\Delta u)+\Big(-\nu\Delta u+\operatorname{curl}(u-\alpha\Delta u)\times u+\nabla\mathfrak{P}\Big)\,dt \\ \quad = F(u,t)\,dt+(u-\alpha\Delta u)\circ\sigma dW, \qquad \text{in } \ \mathcal{O}\times(0,T], \\ \operatorname{div} u=0 \quad \text{in } \mathcal{O}\times(0,T]; \\ u=0 \qquad \text{in } \partial\mathcal{O}\times[0,T]; \\ u(0)=\xi \quad \text{in } \mathcal{O}, \end{cases} \tag{1.1}$$

where $\mathcal{O}$ is a bounded domain of $\mathbb{R}^2$, simply-connected and open, with boundary $\partial\mathcal{O}$ of class $\mathcal{C}^{3,1}$. $u=(u_1,u_2)$ and $\mathfrak{P}$ represent the random velocity and modified pressure, respectively. α,σ are positive constants and ν is the kinematic viscosity. W is a one-dimensional standard Brownian motion defined on a complete filtered probability space $(\Omega,\mathcal{F},P)$ with the augmented Brownian filtration $\{\mathcal{F}_t\}_{t\geq 0}$. ξ is an $\mathcal{F}_T$-measurable random

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variable. The fluid is driven by external forces $F(u,t)\,dt$ and the noise $(u-\alpha\Delta u)\circ\sigma dW$, where the stochastic integral is understood in the sense of anticipating Stratonovich integrals.

We refer the reader to [6–8,4,5] for a comprehensive theory of the second grade fluids. These fluids are non-Newtonian fluids of differential type, they are admissible models of slow flow fluids such as industrial fluids, slurries, polymer melts, etc. They also have interesting connections with other fluid models, see [1,3,2]. For researches on stochastic models of 2D second grade fluids, we refer to [12,13,15,17,16,14].

The consideration of the anticipating initial value is based on several aspects: random measurement errors, the stationary point of the stochastic dynamical system, substitution formulas of anticipating Stratonovich integrals. For more details, we refer to Mohammed and Zhang [9]. The difficulty in directly proving such a substitution theorem is that Kolmogorov continuity theorem fails within our infinite-dimensional setting. To solve this anticipating problem (1.1), we proceed with the following steps: firstly, we develop a simple chain rule of Malliavin derivative of Hilbert space-valued random variables and establish a product rule for the Skorohod integrals, see Lemma 4.1 and Proposition 4.1; secondly, we use Galerkin approximations to show that the solution of (1.1) with deterministic initial value is Mallivin differentiable, see Proposition 4.2; finally, combining the previous two steps, we easily obtain our main results. We believe that this method can also be used to solve the problem with anticipating initial value and linear multiplicative noise for more general framework of SPDE.

The organization of this paper is as follows. In Section 2, we introduce some preliminaries and notations. In Section 3, we formulate the hypotheses and state our main results. Section 4 is devoted to the proof of the main results.

Throughout this paper, $C, C(T), C(T,N)$... are positive constants depending on some parameters $T, N, ...$, whose value may be different from line to line.

2. Preliminaries

In this section, we will introduce some functional spaces, preliminaries and notations.

For $p \geq 1$ and $k \in \mathbb{N}$, we denote by $L^p(\mathcal{O})$ and $W^{k,p}(\mathcal{O})$ the usual L^p and Sobolev spaces over $\mathcal{O}$ respectively, and write $H^k(\mathcal{O}) := W^{k,2}(\mathcal{O})$. We write $\mathbb{X} = X \times X$ for any vector space X. The set of all divergence free and infinitely differentiable functions in $\mathcal{O}$ is denoted by $\mathcal{C}$. $\mathbb{V}$ (resp. $\mathbb{H}$) is the completion of $\mathcal{C}$ in $\mathbb{H}^1(\mathcal{O})$ (resp. $\mathbb{L}^2(\mathcal{O})$). Let $((u,v)) := \int_{\mathcal{O}} \nabla u \cdot \nabla v dx$, where ∇ is the gradient operator. Denote $\|u\| := ((u,u))^{\frac{1}{2}}$. We endow the space $\mathbb{V}$ with the norm $|\cdot|_{\mathbb{V}}$ generated by the following inner product

$$(u,v)_{\mathbb{V}} := (u,v) + \alpha((u,v)), \quad \text{for any } u,v \in \mathbb{V},$$

where $(\cdot,\cdot)$ is the inner product in $\mathbb{L}^2(\mathcal{O})$ (in $\mathbb{H}$). We also introduce the following space

$$\mathbb{W} := \big\{u \in \mathbb{V} : \mathrm{curl}(u - \alpha\Delta u) \in L^2(\mathcal{O})\big\},$$

and endow it with the semi-norm $|\cdot|_{\mathbb{W}}$ generated by the scalar product

$$(u,v)_{\mathbb{W}} := \big(\mathrm{curl}(u-\alpha\Delta u), \mathrm{curl}(v-\alpha\Delta v)\big).$$

In fact, $\mathbb{W} = \mathbb{H}^3(\mathcal{O}) \cap \mathbb{V}$, and this semi-norm $|\cdot|_{\mathbb{W}}$ is equivalent to the usual norm in $\mathbb{H}^3(\mathcal{O})$, the proof can be found in [5,4].