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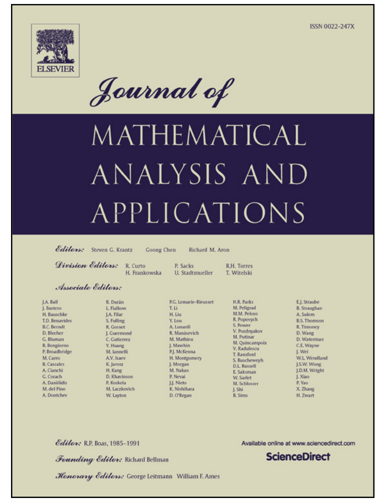

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ON THE UNIQUENESS OF INVERSE SPECTRAL PROBLEMS ASSOCIATED WITH INCOMPLETE SPECTRAL DATA

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Abstract. The inverse spectral problem for a Sturm-Liouville problem which consists of a Sturm-Liouville equation defined on the interval $[0, a]$ and two separated boundary conditions at the endpoints 0 and a, one of which is involved function $f(\lambda)$, is considered in this paper. When $f(\lambda)$ is the type of Herglotz functions and is known as a priori, we prove that the potential on $[0, a]$ and the other boundary condition can be uniquely determined in terms of appropriate partial information on the spectrum and partial information on the set of normalizing constants.

1. Introduction

In recent years there has been considerable interest in the inverse spectral problems with partial information given on the potential q for the following Sturm-Liouville problem:

$$-y'' + qy = \lambda y \tag{1.1}$$

on the unit interval $[0, 1]$ with the boundary conditions:

$$y'(0) - h_0 y(0) = 0, \tag{1.2}$$

$$y'(1) + h_1 y(1) = 0. \tag{1.3'}$$

Here $q \in L^1[0, 1]$, h_0 and h_1 all are real. As was well known, Hochstadt and Lieberman [21] in 1978 first established the so-called half-inverse spectral theorem for the above problem, which says that the whole spectrum σ of Problem (1.1)-(1.3′) determines the potential q uniquely on $[0, 1]$ provided the q is already known on $[0, 1/2]$. Gesztesy and Simon [18] in 1999 gave an important generalization of the half-inverse spectral theorem. They considered the case where q is known on a large interval $[0, a]$ with $a \in [1/2, 1)$ and proved that a certain part of the spectrum σ completely determine q on $[0, a]$, e.g., so do the spectrum, half the spectrum and q on $[1/4, 1]$. In [16], del Rio, Gesztesy and Simon further studied the case where a can be any number in the interval $(0, 1)$, in other words, they considered the problem of uniquely determining the potential q if q is only known on part of the interval $(0, 1)$ (which may be a subinterval of $(0, 1/2)$). These results have been generalized and improved in a variety of ways (see for example [2–4, 22, 23, 30–32] and the references therein). In particular, Wei and Xu in [30], showed that norming constants play an equal role as eigenvalues.

Let $u_+(x, \lambda)$ be the solution of equation (1.1) satisfying the initial conditions $u_+(1) = 1$ and $u'_+(1) = -h_1$. We observe that if the potential q is known a priori

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