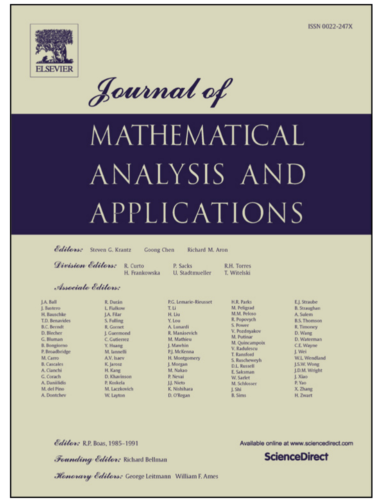
Accepted Manuscript



Global boundedness in a fully parabolic quasilinear chemotaxis system with singular sensitivity

Mengyao Ding

PII: S0022-247X(18)30079-9
DOI: https://doi.org/10.1016/j.jmaa.2018.01.049
Reference: YJMAA 21982

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 3 October 2017

Global boundedness in a fully parabolic quasilinear chemotaxis system with singular sensitivity*

Mengyao Ding†

School of Mathematical Sciences, Peking University, Beijing 100089, P.R. China

January 25, 2018

Abstract. In this paper we study the global boundedness of solutions to the quasilinear fully parabolic chemotaxis system: $u_t = \nabla \cdot (D(u)\nabla u - S(u)\nabla\varphi(v))$, $v_t = \Delta v - v + u$, where bounded domain $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ subject to the non-flux boundary conditions, the diffusivity fulfills $D(u) = a_0(u+1)^{-\alpha}$ with $a_0 > 0$ and $\alpha \geq 0$, while the density-signal governed sensitivity satisfies $0 \leq S(u) \leq b_0(u+1)^{\beta}$ and $0 < \varphi'(v) \leq \frac{\chi}{v^k}$ for $b_0, \chi > 0$ and $\beta, k \in \mathbb{R}$. It is shown that the solution is globally bounded provided $\alpha + \beta < 1$ and $k \leq 1$. This result demonstrates the effect of signal-dependent sensitivity on the blow-up prevention.

Keywords: Chemotaxis; Nonlinear diffusion; Global boundedness; Signal-dependent sensitivity

Mathematics Subject Classification: 92C17; 35K55; 35B35; 35B40

1 Introduction

In this article, we study the following parabolic-parabolic Keller-Segel system

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \nabla \cdot (S(u)\nabla\varphi(v)), & x \in \Omega,\ t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega,\ t > 0, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), v(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ $(n \geq 2)$ with smooth boundary, $\frac{\partial}{\partial \nu}$ denotes the derivative with respect to the outer normal of $\partial\Omega$. u and v stand for the cell density, the concentration of an attractive signal produced by cells themselves respectively. Initial data u_0 and v_0 fullfill

$$\begin{cases} u_0 \in C(\bar{\Omega}), \quad u_0 \geq 0 \text{ in } \bar{\Omega}, \quad u_0 \not\equiv 0, \\ v_0 \in W^{1,\infty}(\Omega), \quad v_0 > 0 \text{ in } \bar{\Omega}. \end{cases} \tag{1.2}$$

*Supported by the National Natural Science Foundation of China (11571020, 11671021)

†Corresponding author. E-Mail: mengyaod@126.com