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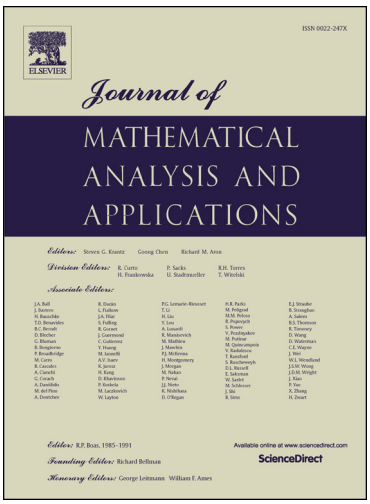

Almost periodic solutions at infinity of differential equations and inclusions

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ALMOST PERIODIC SOLUTIONS AT INFINITY OF DIFFERENTIAL EQUATIONS AND INCLUSIONS

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Abstract. We study the notion of a function almost periodic at infinity and present some spectral conditions of the almost periodicity at infinity for bounded solutions of a differential inclusions governed by closed multivalued linear operators in Banach spaces. Applications to degenerate differential equations and semilinear differential inclusions are given. As example, we consider the problem of stabilization of solutions to a heat equation.

1. Introduction

The problem of almost periodicity of bounded solutions to various classes of differential equations on the real axis $\mathbb{R} = (-\infty, +\infty)$ and the half-axis $\mathbb{R}_+ = [0, +\infty)$ was studied by many authors (see [1], [2], [5], [8], [9], [10], [23], [25], [26], [27], [28], [29], [30], [33]). One of the first works of this direction was the paper of S. Bochner and J. Von Neumann [20] in which the almost periodicity of bounded solutions of difference equations was established. In the papers of S.L. Sobolev [32] criteria of almost periodicity of solutions to parabolic equations with a self-adjoint operator in a Hilbert space were obtained.

The basis of these numerous studies is the spectral criterion of L.H. Loomis [29] of almost periodicity of a function (see also its vector analog [9], [10]) as well as the theorem on the Beurling spectrum of bounded solutions of differential equations obtained in [9], [10] (its version is presented in Theorem 5 below.

In the papers [33], [34], [30] spectral criteria for asymptotic almost periodicity of solutions to differential equations were obtained. These studies were continued in the works [7], [8], [31], [3], [4]. Notice that the assumption on the operator coefficient of the equation of being the infinitesimal generator of a C_0-semigroup of bounded operators is omitted in [4] (see Corollary 1 of Theorem 6).

Usually in these studies, various classes of linear equations with almost periodic (in the sense of Bohr) coefficients and an almost periodic right-hand side were considered. However, it is worth noting that a bounded solution of a simplest differential equation in a finite-dimensional space of the form $x'(t) = Ax(t) + \psi(t)$, $t \geq 0$, where A is a linear operator and ψ is a continuous function, decreasing at infinity is not almost periodic in the usual sense.

In the papers of the first author [13], [14] a new class of continuous functions, named almost periodic at infinity was introduced. This class includes into itself

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