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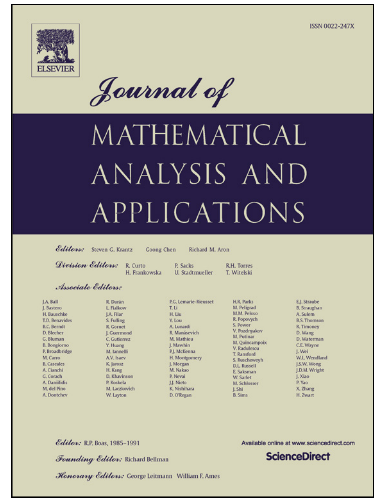

On operators with spectrum in Cantor sets and spectral synthesis

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ACCEPTED MANUSCRIPT

ON OPERATORS WITH SPECTRUM IN CANTOR SETS AND SPECTRAL SYNTHESIS.

MOHAMED ZARRABI

Abstract. In this paper we investigate the behaviour of powers of operators with spectrum contained in the Cantor sets. As consequence we show that the Cantor sets satisfy spectral synthesis in a large class of weighted Beurling algebras.

1. Introduction.

Let T be an invertible operator defined on a Banach space with spectrum ($\mathrm{Sp}(T)$) contained in the unit circle $\mathbb{T}$. Several results show that growth conditions on $(\|T^n\|)_{n\in\mathbb{Z}}$ and on the size of the spectrum of T imply stronger properties on T. It is proved by I. M. Gelfand in [11] that if $\sup_{n\in\mathbb{Z}} \|T^n\| < +\infty$ and if $\mathrm{Sp}(T) = \{1\}$ then $T = I$, where I is the identity operator. The author shows in [16], that if T is a contraction such that $\mathrm{Sp}(T)$ is countable and

$$\text{for all } \epsilon > 0, \quad \|T^{-n}\| = O(e^{\epsilon\sqrt{n}}), n \to +\infty,$$

then T is an isometry. See also [6, Theorem 5.6] for a similar result on representations of groups by operators.

For $\xi \in \left(0, \frac{1}{2}\right)$, let E_ξ be the perfect symmetric set associated with ξ, that is

$$E_\xi = \left\{ \exp\left(2i\pi(1-\xi)\sum_{n=1}^{+\infty} \epsilon_n \xi^{n-1}\right) : \epsilon_n = 0 \text{ or } 1 \quad (n \geq 1)\right\},$$

and let

$$b(\xi) = \frac{\log\frac{1}{\xi} - \log 2}{2\log\frac{1}{\xi} - \log 2}.$$

Notice that $E_{1/3}$ is the classical triadic Cantor set. Let $q \geq 3$ be an integer. J. Esterle showed in [9, p. 79] that if

$$\sup_{n\geq 0} \|T^n\| < +\infty,$$

$$\left\|T^{-n}\right\| = O\left(e^{n^\beta}\right),\ n \to +\infty,\ \text{for some } \beta < b(1/q),$$

and if $\mathrm{Sp}(T) \subset E_{1/q}$, then T satifies the condition

$$\sup_{n\geq 0} \|T^{-n}\| < +\infty.$$

On the other hand it is proved in [10, Theorem 2.2] that the constant $b(1/q)$ is sharp. For further results see [1], [2], [4], [7], [8], [9], [16]. In this paper we are interested by operators

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