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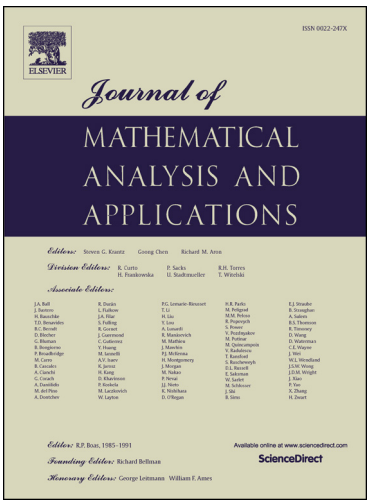

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A complete general solution of the unsteady Brinkman equations

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Abstract

In this paper, we present a complete general solution of the unsteady Brinkman equations. To this end, we introduce a representation for velocity and pressure in terms of two scalar functions. One of these scalar functions satisfies a second order partial differential equation (PDE) while the other satisfies a fourth order PDE which can be factorized into a pair of second order PDEs. We show that the solution of this fourth order PDE is indeed the sum of the solutions of the two second order PDEs. We also use these solutions to obtain a complete general solution of the unsteady Brinkman equations.

Keywords: Stokes flows, Brinkman equations, heat operator, factorization of operators.
2008 MSC: 35Q35, 76D07.

1. Introduction

Consider a homogeneous porous medium characterized by a permeability parameter κ/L^2, where κ is the Darcy permeability of the medium and L is the particle length scale of a cloud of spherical particles in the medium. This kind of a medium is said to be a Brinkman medium. Moreover, in a Brinkman medium the size of the particles is smaller than the characteristic length scale of the flow. Therefore, they occupy a negligible volume (see [5]). Flows through porous media are governed by either the Darcy model or the Brinkman model. The latter is found to be more suitable when the permeability of the medium is high. Consider an incompressible flow of a viscous fluid through a Brinkman medium with a permeability κ which is modeled by the Brinkman equations,

$$\rho\frac{\partial \mathbf{q}}{\partial t}(\mathbf{x},t) = -\nabla p(\mathbf{x},t) + \mu\left(\Delta - \frac{1}{\kappa}\right)\mathbf{q}(\mathbf{x},t), \quad \mathbf{x}\in\mathbb{R}^3,\ t>0, \tag{1}$$

$$\operatorname{div}(\mathbf{q}(\mathbf{x},t)) = 0, \quad \mathbf{x}\in\mathbb{R}^3,\ t>0. \tag{2}$$

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