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Upper semicontinuity of the pullback attractors of non-autonomous damped wave equations with terms concentrating on the boundary

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A B S T R A C T

In this paper we analyze the asymptotic behavior of the pullback attractors of a non-autonomous damped wave equation when some reaction terms are concentrated in a neighborhood of the boundary and this neighborhood shrinks to boundary as a parameter ε goes to zero. We prove a result of regularity of the pullback attractors and that this family of attractors is upper semicontinuous at $\varepsilon = 0$.

1. Introduction

In this work, we analyze the asymptotic behavior of the pullback attractors of a non-autonomous damped wave equation when some reaction terms are concentrated in a neighborhood of the boundary and this neighborhood shrinks to boundary as a parameter ε goes to zero.

To describe the problem, let Ω be an open bounded smooth set in $\mathbb{R}^3$ with a smooth boundary $\Gamma = \partial\Omega$. We define the strip of width ε and base $\partial\Omega$ as

$$\omega_\varepsilon = \{x - \sigma \vec{n}(x) : \ x \in \Gamma \ \text{ and } \ \sigma \in [0, \varepsilon)\},$$

for sufficiently small ε, say $0 < \varepsilon \leqslant \varepsilon_0$, where $\vec{n}(x)$ denotes the outward normal vector at $x \in \Gamma$. We note that the set ω_ε has Lebesgue measure $|\omega_\varepsilon| = O(\varepsilon)$ with $|\omega_\varepsilon| \leqslant k\,|\Gamma|\,\varepsilon$, for some $k > 0$ independent of ε, and that for small ε, the set ω_ε is a neighborhood of Γ in $\bar{\Omega}$, that collapses to the boundary when the parameter ε goes to zero (see Fig. 1).

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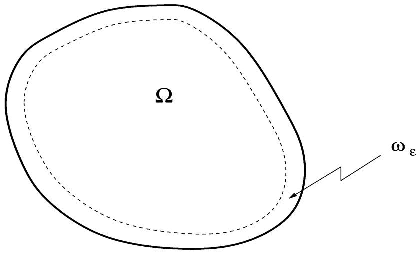

Fig. 1. The set ω_ϵ.

We are interested in the behavior, for small ε, of the solutions of the non-autonomous damped wave equation with concentrated terms given by

$$\begin{cases} u^\varepsilon_{tt} - \operatorname{div}(a(x)\nabla u^\varepsilon) + u^\varepsilon + \beta(t)u^\varepsilon_t = f(u^\varepsilon) + \dfrac{1}{\varepsilon}\chi_{\omega_\varepsilon} g(u^\varepsilon) & \text{in } \Omega \times (\tau, +\infty), \\ a(x)\dfrac{\partial u^\varepsilon}{\partial \vec{n}} = 0 & \text{on } \Gamma \times (\tau, +\infty), \\ u^\varepsilon(\tau) = u_0 \in H^1(\Omega), \quad u^\varepsilon_t(\tau) = v_0 \in L^2(\Omega), \end{cases} \tag{1.1}$$

where $a \in \mathcal{C}^1(\overline{\Omega})$ with

$$0 < a_0 \leqslant a(x) \leqslant a_1, \quad \forall x \in \overline{\Omega}, \tag{1.2}$$

for fixed constants $a_0, a_1 > 0$, and $\chi_{\omega_\varepsilon}$ denotes the characteristic function of the set ω_ε. We refer to $\dfrac{1}{\varepsilon}\chi_{\omega_\varepsilon} g(u^\varepsilon)$ as the concentrating reaction in ω_ε. We assume that $\beta : \mathbb{R} \to \mathbb{R}$ is bounded, globally Lipschitz, and

$$\beta_0 \leqslant \beta(t) \leqslant \beta_1, \quad \forall t \in \mathbb{R}, \tag{1.3}$$

for fixed constants $\beta_0, \beta_1 > 0$ (the assumption that β is globally Lipschitz continuity can be weakened to uniform continuity on $\mathbb{R}$ and continuous differentiability).

We take $f, g : \mathbb{R} \to \mathbb{R}$ to be $\mathcal{C}^2$ and assume that it satisfies the growth estimates

$$|j'(s)| \leqslant c(1 + |s|^{\rho_j}), \quad \forall s \in \mathbb{R}, \tag{1.4}$$

and

$$|j(s_1) - j(s_2)| \leqslant c|s_1 - s_2|(1 + |s_1|^{\rho_j} + |s_2|^{\rho_j}), \quad \forall s_1, s_2 \in \mathbb{R}, \tag{1.5}$$

with $j = f$ or $j = g$ and exponents ρ_f and ρ_g, respectively, such that $\rho_f \leqslant 2$ and $\rho_g \leqslant 1$. We note that the estimate (1.4) implies (1.5).

Moreover, we assume the growth estimate

$$|j''(s)| \leqslant c, \quad \forall s \in \mathbb{R}, \tag{1.6}$$

also we assume that

$$\limsup_{|s| \to +\infty} \frac{j(s)}{s} \leqslant 0, \tag{1.7}$$

with $j = f$ or $j = g$. We note that (1.7) is equivalent to saying that for any $\gamma > 0$ there exists $c_\gamma > 0$ such that

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