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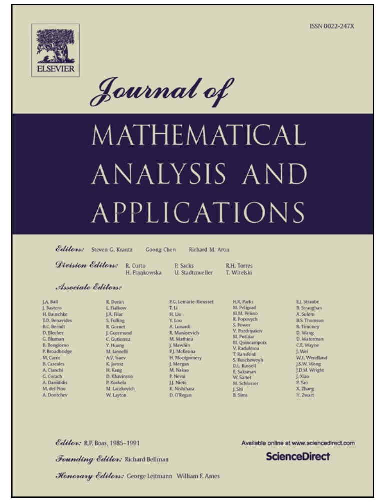
Dorin Ervin Dutkay, Rajitha Ranasinghe



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WEIGHTED FOURIER FRAMES ON SELF-AFFINE MEASURES

DORIN ERVIN DUTKAY AND RAJITHA RANASINGHE

ABSTRACT. Continuing the ideas from our previous paper [8], we construct Parseval frames of weighted exponential functions for self-affine measures.

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1. Introduction

A probability measure μ on $\mathbb{R}$ is called *spectral* if there exists a sequence of exponential functions which form an orthonormal basis for $L^2(\mu)$. Of course, the main example is the Lebesgue measure on the unit interval with the classical Fourier series. In 1998, Jorgensen and Pedersen [10] constructed the first example of a singular, non-atomic spectral measure, based on a Cantor set with scale 4. Since then, many other examples of spectral singular measures have been constructed (see e.g., [14, 11, 3, 4]), most of them are based on affine iterated function systems (see Definition 1.1). In the same paper, Jorgensen and Pedersen showed that the Hausdorff measure on the Middle Third Cantor set is not spectral and Strichartz [14] posed the question whether there are any frames of exponential functions for the Middle Third Cantor set. As far as we know, this question is still open.

In search of a frame for the Middle Third Cantor set, in [12], Picioroaga and Weber introduced an interesting idea for the construction of weighted exponential frames (also called weighted Fourier frames) for the self-affine measures, in particular for the Cantor set C_4 in Jorgensen and Pedersen's example. The word "weighted" means that the exponential function is multiplied by a constant. The basic idea is to use Cuntz algebras to construct an orthonormal set for a dilation of the Hilbert space of the fractal measure, which then projects into a Parseval frame of weighted exponential functions. In [8], the authors generalized the aforementioned idea of Picioroaga and Weber to construct Parseval Fourier frames for self-affine measures (see Definition 1.1).

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