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Stability analysis on a type of steady state for the SKT competition model with large cross diffusion

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ABSTRACT

This paper is concerned with the existence and the stability of steady state solutions for the SKT biological competition model with cross-diffusion. By applying the higher order expansion and some detailed spectral analysis to the limiting system as the cross diffusion rate tends to infinite, it is proved that the nontrivial positive steady states with some special bifurcating structure are unstable. Further, the existence and the instability of the corresponding nontrivial positive steady states for the original cross-diffusion system are proved by applying perturbation argument. Finally, we show the global existence of solutions for the limiting system and present some numerical simulation on the large time behavior of the solution with more general initial data.

1. Introduction and statement of main results

In the field of population dynamics, some models of the multi-species interacting populations are described by reaction–diffusion systems. In order to investigate the spatial segregation under inter- and intra-population pressure, Shigesada et al. [21] proposed a two-species competition model with self- and cross-diffusion, which is described by the following S–K–T competition model:

$$\begin{cases} u_t = \Delta[(d_1+\rho_{11}u+\rho_{12}v)u] + u(a_1-b_1u-c_1v), & x\in\Omega, t>0,\\ v_t = \Delta[(d_2+\rho_{21}u+\rho_{22}v)v] + v(a_2-b_2u-c_2v), & x\in\Omega, t>0,\\ \frac{\partial u}{\partial\nu} = \frac{\partial v}{\partial\nu} = 0, & x\in\partial\Omega, t>0,\\ u(x,0)=u_0(x)\ge 0, v(x,0)=v_0(x)\ge 0, & x\in\ \Omega, \end{cases} \tag{1.1}$$

where $u(x,t)$ and $v(x,t)$ represent the densities of two competing species at location x and time t. Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$ and ν is the outward unit normal vector on $\partial\Omega$. We assume d_i, a_i, b_i, c_i are positive constants throughout this paper. The coefficients ρ_{11} and ρ_{22} are the

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self-diffusion rates which represent intra-specific population pressures, ρ_{12} and ρ_{21} denote the cross-diffusion coefficients which measure the population pressure from the competing species.

The S–K–T model with cross-diffusion has attracted tremendous attention of both mathematicians and ecologists in recent two decades. For the S–K–T model in bounded domain, the existence of global solution in time, the existence and the stability/instability of steady states have been widely investigated (see [4], [8], [10], [12–17,19,20], [22–26] and the references therein). Especially for the one dimensional case and $\rho_{11} = \rho_{21} = \rho_{22} = 0$ but $\rho_{12} > 0$, the system (1.1) is simplified as

$$\begin{cases} u_t = [(d_1 + \rho_{12} v)u]_{xx} + u(a_1 - b_1 u - c_1 v), & x \in (0, 1),\ t > 0, \\ v_t = d_2 v_{xx} + v(a_2 - b_2 u - c_2 v), & x \in (0, 1),\ t > 0, \\ u_x = v_x = 0, & x = 0,\ 1,\ t > 0, \end{cases} \tag{1.2}$$

for which there are some interesting results on the existence and the stability of the positive steady states when ρ_{12} is not small. It was shown in [19] that for each fixed $d_2 > 0$ small enough, when d_1 and $\frac{\rho_{12}}{d_1}$ are large enough there exist positive steady states with interior or boundary transition layers for (1.2). The stability/instability of such steady states with transition layers was proved in [8] by applying SLEP method. Lou and Ni [14] showed that for small $d_2 > 0$, when ρ_{12} (or d_1) and $\frac{\rho_{12}}{d_1}$ are large enough, there exist two types of positive spiky steady states with finite height of boundary layer. The instability of these two types of spiky steady states were proved in [23] and [22].

In [14], the authors also proposed two types of limiting stationary problem of (1.1), which describe the limiting characterization of the steady states as ρ_{12} tends to infinity (see Theorem 1.4 and 4.1 in [14]). For convenience of our later use, we shall restate the related results in [14] for the special case $\rho_{11} = \rho_{21} = \rho_{22} = 0$ as follows.

Theorem 1. *[14] Let $\rho_{11} = \rho_{21} = \rho_{22} = 0$, and suppose that $n \le 3$, $\frac{b_1}{b_2} \ne \frac{a_1}{a_2} \ne \frac{c_1}{c_2}$ and $\frac{a_2}{d_2}$ is not equal to any eigenvalue of $-\Delta$ with homogeneous boundary condition on $\partial\Omega$. Let (u_i, v_i) be a positive nontrivial steady state of (1.1) with $\rho_{12} = \rho_{12,i}$. Then, as $\rho_{12,i} \to \infty$, by passing to a subsequence if necessary, (u_i, v_i) must satisfy either (i) or (ii) below.*

(i) $(u_i, \rho_{12,i} v_i)$ converges uniformly to (u, w), which satisfies

$$\begin{cases} \Delta[(d_1 + w)u] + u(a_1 - b_1 u) = 0, & x \in \Omega, \\ d_2 \Delta w + w(a_2 - b_2 u) = 0, & x \in \Omega, \\ \frac{\partial u}{\partial \nu} = \frac{\partial w}{\partial \nu} = 0, & x \in \partial\Omega. \end{cases} \tag{1.3}$$

(ii) (u_i, v_i) converges uniformly to $(\frac{\zeta}{\psi}, \psi)$, where $\zeta > 0$ is a constant and $(\psi(x), \zeta)$ satisfies

$$\begin{cases} d_2 \Delta\psi + \psi(a_2 - c_2 \psi) - b_2 \zeta = 0, & x \in \Omega, \\ \frac{\partial \psi}{\partial \nu} = 0, & x \in \partial\Omega, \\ \int_\Omega \frac{1}{\psi}(a_1 - \frac{b_1 \zeta}{\psi} - c_1 \psi) = 0. \end{cases} \tag{1.4}$$

In [16], the authors obtained some nearly optimal results on the existence and non-existence of the positive steady states for the second limiting system (shadow system) (1.4) in the one dimensional case, and in [16] it was also proved that there exist some new types of nonconstant steady states including the large spiky steady states as $d_2 \to 0$ and the nontrivial positive steady states with some singular structures as $d_2 \to a_2/\pi^2$ for the shadow system (1.4). When d_2 is small enough, the existence and the detailed structure of the large spiky steady states for the shadow system (1.4) and for the original cross-diffusion system were obtained in [24]. When d_2 is near a_2/π^2, the detailed structure and the stability of the nontrivial steady states for the shadow system (1.4) and for the original cross-diffusion system were proved in [20]. Recently,