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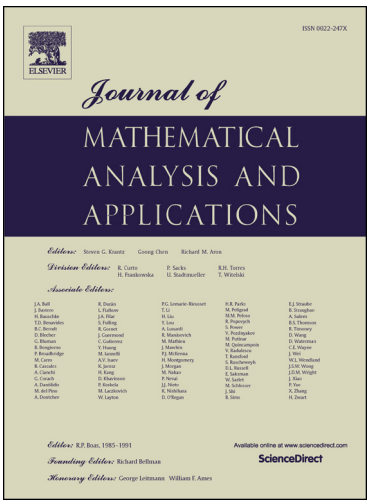

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Zhaowen Zheng, Qingkai Kong

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Friedrichs Extensions for Singular Hamiltonian Operators with Intermediate Deficiency Indices

Zhaowen Zheng[1]*and Qingkai Kong[2]

1. School of Mathematical Sciences,

Qufu Normal University, Qufu 273165, Shandong, P. R. China

E-mail: zhwzheng@126.com

2. Department of Mathematics, Northern Illinois University,

DeKalb, IL 60115, USA

E-mail: qkong@niu.edu

ABSTRACT For singular Hamiltonian operators in the intermediate deficiency indices case, we give a complete characterization of Friedrichs extensions of minimal Hamiltonian operators, which unifies and generalizes some known results in the literature. The exact boundary conditions for the Friedrichs extensions are constructed via the principal solutions. The main approach in this paper is the Friedrichs construction by way of the refined LC-type solutions at singular endpoints.

KEYWORDS Hamiltonian operator; Friedrichs extension; LC-type solution; intermediate deficiency index; disconjugate.

AMS (MOS) subject classification Primary 34B20 Secondary 34L05, 47E05, 47B25.

1 Introduction

For any bounded below symmetric operator S which is densely defined (not necessarily closed) in a Hilbert space $(\mathbb{H}, \langle\cdot,\cdot\rangle)$, there always exists a self-adjoint extension which preserves the lower bound. This self-adjoint extension is known as Friedrichs extension. In order to obtain such an extension, one can use the Friedrichs construction or its variantion. Suppose

$$\langle Sy, y\rangle = \langle y, Sy\rangle \geq \gamma\langle y, y\rangle, \quad y \in \mathcal{D}(S)$$

for some constant $\gamma \in \mathbb{R}$. Then the domain $\mathcal{D}(S_F)$ of the Friedrichs extension S_F is the collection of the functions y which satisfy the following conditions: there exists $\{y_n\} \subset \mathcal{D}(S)$ such that

$$\begin{aligned} &(i) \qquad y_n \to y \quad \text{in} \ \ \mathbb{H}, \\ &(ii) \qquad \langle S(y_n - y_m), y_n - y_m\rangle \to 0, n, m \to \infty. \end{aligned}$$

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