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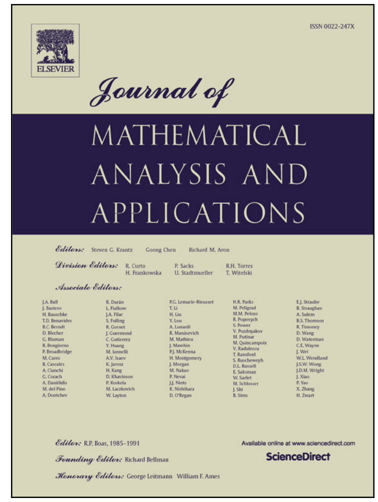

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ON THE CONJECTURE OF WOOD AND PROJECTIVE HOMOGENEITY

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Abstract. In 2005 Kawamura and Rambla, independently, constructed a metric counterexample to Wood's Conjecture from 1982. We exhibit a new nonmetric counterexample of a space $\hat{L}$, such that $C_0(\hat{L}, \mathbb{C})$ is almost transitive, and show that it is distinct from a nonmetric space whose existence follows from the work of Greim and Rajagopalan in 1997. Up to our knowledge, this is only the third known counterexample to Wood's Conjecture. We also show that, contrary to what was expected, if a one-point compactification of a space X is R.H. Bing's pseudo-circle then $C_0(X, \mathbb{C})$ is not almost transitive, for a generic choice of points. Finally, we point out close relation of these results on Wood's conjecture to a work of Irwin and Solecki on projective Fraïssé limits and projective homogeneity of the pseudo-arc and, addressing their conjecture, we show that the pseudo-circle is not approximately projectively homogeneous.

1. Introduction

In 1982 G.V. Wood stated the following conjecture, in the isometric theory of Banach spaces.

Wood's Conjecture, [39]. *Suppose L is a locally compact Hausdorff space such that the space of all scalar-valued functions vanishing at infinity $C_0(L, \mathbb{K})$, equipped in the supremum norm, is almost transitive. Then L consists of a single point.*

Wood's conjecture is related to Banach-Mazur rotation problem, which asks whether a separable Banach space with a transitive norm has to be isometric or isomorphic to a Hilbert space. Recall that a Banach space $(Y, ||\cdot||)$ is called *almost transitive* if for any $\epsilon > 0$ and any $y_1, y_2 \in Y$ with $||y_1|| = ||y_2|| = 1$ there exists a surjective linear isometry T such that $||Ty_1 - y_2|| < \epsilon$, and it is called *transitive* if for such y_1 and y_2 there exists a surjective linear isometry T with $Ty_1 = y_2$. As a consequence of Banach-Stone Theorem, Wood's Conjecture is a topological problem. In 1997 Greim and Rajagopalan [14] proved Wood's Conjecture in the real case; i.e. if $C_0(L, \mathbb{R})$ is almost transitive then L is a singleton. They also showed that the existence of a counterexample in the complex case implies the existence of a nonmetric locally compact Hausdorff space $\tilde{L}$ such that $C_0(\tilde{L}, \mathbb{C})$ is transitive. In 2005 Kawamura and Rambla, independently, disproved Wood's conjecture.

Theorem 1.1 (Kawamura [20], Rambla [36])**.** *If X is a pseudo-arc and $p \in X$ then for $L = X \setminus \{p\}$ the space $C_0(L, \mathbb{C})$ is almost transitive.*