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Asymptotic boundary behavior of the Bergman curvatures of a pseudoconvex domain ☆

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A B S T R A C T

We present a method of obtaining a lower bound estimate of the curvatures of the Bergman metric without using the regularity of the kernel function on the boundary. As an application, we prove the existence of an uniform lower bound of the bisectional curvatures of the Bergman metric of a smooth bounded pseudoconvex domain near the boundary with constant Levi rank.

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1. Introduction

The curvatures of the Bergman metric, such as holomorphic sectional curvatures, bisectional curvatures and Ricci curvatures are important invariants in complex differential geometry. It is well-known that the bisectional curvatures (holomorphic sectional curvatures) and the Ricci curvatures of the Bergman metric of an n-dimensional complex manifold are always less than 2 and $n+1$, respectively (see [13]). On the contrary, there exists an example constructed by Herbort [7], for which the holomorphic sectional curvature is not bounded from below in certain direction. To obtain a lower bound estimate of the curvatures of the Kähler metric from a direct calculation, estimates of derivatives of the potential function up to order 4 are necessary. Therefore, in the case of the Bergman metric, estimates of derivatives of the Bergman kernel function are needed.

For instance, Klembeck [12] showed that the holomorphic sectional curvatures of a C^∞-smooth strongly pseudoconvex bounded domain in $\mathbb{C}^n$ approach $-4/(n+1)$ near the boundary by using Fefferman's asymptotic formula for the Bergman kernel function. Even though an asymptotic formula for the Bergman kernel function does not exist, McNeal proved in [15] that the holomorphic sectional curvature of the Bergman

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metric for a pseudoconvex bounded domain in $\mathbb{C}^2$ with finite type boundary has to be bounded. He used the subelliptic estimate for the $\overline{\partial}$-Neumann problem which gives information on the regularity of the kernel function up to the boundary.

Moreover, Kim and Yu generalized in [11] Klembeck's theorem to the case of C^2-smooth boundary, without using Fefferman's asymptotic formula. Therefore, it is natural to ask whether one can prove McNeal's result without using the results on the $\overline{\partial}$-Neumann problem. This was posed as Problem 26 in [5].

In this paper, we give an answer to this problem. The precise result is the following theorem.

Theorem 1.1. *Let $\Omega \subset\subset \mathbb{C}^n$ be a pseudoconvex domain with smooth boundary. If the Levi form has constant rank l, $0 \le l \le n-1$ on a neighborhood V of $z_0 \in \partial\Omega$, then there is a neighborhood W of z_0 such that all the bisectional curvatures of the Bergman metric of Ω are bounded below by a negative constant on W. In particular, the holomorphic sectional curvatures and Ricci curvatures are also bounded below.*

Notice that if the Levi form is identically zero, then we cannot guarantee the regularity of the kernel function on the boundary since the pseudo-local property of Neumann operator does not hold (cf. [9], [14]).

The idea of the proof of Theorem 1.1 is as follows: since the bisectional curvatures can be represented by Bergman's minimum integrals (Theorem 2.1), we only need to get lower and upper bound estimates of the minimum integrals. Lower bounds are easily obtained by the minimum integrals of a subdomain. To achieve an upper bound, we apply Hörmander's L^2-estimates of $\overline{\partial}$ with a plurisubharmonic function with large Hessians on the subdomain, which is constructed by Fu in [6]. The same argument with the result of Catlin [2] gives an answer of Problem 26 in [5]. Our result is an improvement upon the results of [15] in this respect, since the holomorphic sectional curvatures does not control the bisectional curvatures and Ricci curvatures in general.

This paper is organized as follows: First, we briefly review fundamentals of Bergman geometry including Bergman's minimum integrals and applications of Hörmander's L^2-estimates of $\overline{\partial}$ to the minimum integrals. In section 4, we review the result of Fu [6] on the construction of a family of plurisubharmonic functions with large Hessians. Then we give a proof of Theorem 1.1 with detail. In the last section, we show that our method is applicable to other pseudoconvex domains beyond the case of constant Levi rank (Theorem 6.1).

2. Preliminaries

Let Ω be a bounded domain in $\mathbb{C}^n$. Define the *Bergman space*

$$\mathcal{A}^2(\Omega) := L^2(\Omega) \cap \mathcal{H}(\Omega),$$

where $\mathcal{H}(\Omega)$ is the space of holomorphic functions on Ω and $L^2(\Omega)$ is the space of square integrable functions on Ω. Let $\{\phi_j\}_{j=0}^{\infty}$ be a complete orthonormal basis for $\mathcal{A}^2(\Omega)$. Then the *Bergman kernel* and *Bergman metric* for Ω are defined by

$$K_\Omega(z,\overline{z}) := \sum_{j=0}^{\infty} \phi_j(z)\overline{\phi_j(z)},$$

$$g_\Omega(z;X) := \sum_{j,k=1}^{n} \frac{\partial^2 \log K_\Omega(z,\overline{z})}{\partial z_j \partial \overline{z_k}} X_j \overline{X_k},$$

where $z \in \Omega$ and $X = \sum_{i=1}^n X_i \frac{\partial}{\partial z_i} \in T_z^{1,0}(\Omega)$.

Since the Bergman metric is a Kähler metric, the *bisectional curvature* $B_\Omega(z;X,Y)$ at z along the directions X and Y is defined by

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