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Extremes on different grids and continuous time of stationary processes

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ABSTRACT

This paper investigates asymptotical distributions of extremes on different grids and continuous time of stationary processes. The findings show that these joint extremes over certain threshold dependent grids are asymptotically completely dependent, dependent and independent respectively for dense, Pickands and sparse grids. Furthermore, an integral representation of the asymptotic function involved in the approximation is obtained.

1. Introduction

Let $\{X(t), t \geq 0\}$ be a stationary process with continuous sample paths. Of primary interest is to investigate extremes $(M(h), M(\delta, h))$ given by

$$M(h) = \max_{t\in[0,h]} X(t) \quad \text{and} \quad M(\delta, h) = \max_{t\in\mathfrak{R}(\delta)\cap[0,h]} X(t), \quad \delta,\, h > 0,$$

where $\mathfrak{R}(\delta) = \{k\delta : k \in \mathbb{N}\}$ is referred to as a uniform grid with step length δ. The dependence structure plays an important role in numerical simulations of high extremes of continuous-time stationary processes. Typical related discussions are referred to [11,19–21] for Gaussian processes; [12] for the storage process with fractional Brownian motion as input; [10,27,26] for vector stationary Gaussian processes and standardized stationary Gaussian processes; and [17,28] for chi-processes and stationary processes. We see from the aforementioned contributions that the asymptotic behavior of extremes on different grids and continuous time is strongly determined by step length δ; see for related discussions [4,22,28] in financial and time series literature.

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This paper concerns continuous-time stationary processes $\{X(t), t \geq 0\}$ with marginal distribution G, and investigates tail asymptotics of extremes on different grids and continuous time. To this end, several mild conditions are required as follows, which are cited from [2].

Condition A: Suppose that X is a stationary process with almost surely continuous sample paths and marginal distribution G in the Gumbel max-domain of attraction (MDA), i.e., there exists a positive scaling function $w(\cdot)$ such that (set below $u_x = u + x/w(u)$)

$$\lim_{u\to\infty} \frac{1-G(u_x)}{1-G(u)} = \mathrm{e}^{-x}, \quad \forall x \in \mathbb{R}. \tag{1}$$

Assume further that there exists for any $x \in \mathbb{R}$ a random process $\{\xi_x(t), t \geq 0\}$ and a strictly positive non-increasing function $q = q(u)$, $u > 0$ with $\lim_{u\to\infty} q(u) = 0$, such that

$$\left\{ w(u)(X(q(u)t) - u) \middle| X(0) > u_x,\, t \geq 0 \right\} \xrightarrow{d} \{\xi_x(t),\, t \geq 0\}, \quad u \to \infty, \tag{2}$$

where $\xrightarrow{d}$ denotes the weak convergence of finite dimensional distributions.

Hereafter, the function $q = q(u)$ involved below is the same as mentioned in condition **A** unless stated otherwise, and the uniform grid $\mathfrak{R}(\delta) = \{k\delta, k \in \mathbb{N}\}$ is the so-called dense, Pickands and sparse if step length $\delta = \delta(u)$ is such that $\delta(u) \leq K$ with some constant $K > 0$ and $\lim_{u\to\infty} \delta(u)/q(u) = a \in (0, \infty)$ with $a = 0, a \in (0, \infty)$ and $a = \infty$, respectively.

Condition B: (Short-lasting-exceedance) For positive constants a and h (set $[x]$ the integer part of x)

$$\lim_{N\to\infty} \limsup_{u\to\infty} \sum_{k=N}^{[h/(aq(u))]} \mathbb{P}\left(X(aq(u)k) > u_x \middle| X(0) > u_x\right) = 0, \quad x \in \mathbb{R}. \tag{3}$$

Condition C*: Suppose for all $h > 0$ and $x \in \mathbb{R}$ that

$$\lim_{a\downarrow 0} \limsup_{u\to\infty} \frac{q(u)}{1-G(u)} \mathbb{P}\left(M(h) > u_x, M(aq(u), h) \leq u_x\right) = 0. \tag{4}$$

We see from [9,14,17,18] that conditions **A**, **B** and **C*** above are satisfied by skew Gaussian processes, chi-processes and order statistics (Gaussian) processes.

This paper aims to investigate tail asymptotics of joint extreme $(M(h), M(\delta, h))$ of a stationary process X with $\delta = \delta(u)$ a Pickands, dense and sparse grid. The crucial methodology is to develop the discretization approximation by [2] to be of joint form. Theorems 2.1, 2.4, 2.7 and 2.6 concern tail asymptotics of joint extremes over fixed time interval, which together with certain global mixing conditions are further applied to establish the Gumbel limit theorem in Theorem 3.2. It turns out that the joint extremes are asymptotically dependent, completely dependent and independent respectively for Pickands, dense and sparse grids. Furthermore, we give an integral representation of the asymptotic function, which is related to the extremal index and Pickands constant, see e.g., [5–8,23].

The rest of the paper is organized as follows. Section 2 is devoted to establishing main results followed by Section 3 discussing the asymptotic function and Gumbel limit theorem. The proofs are relegated to Section 4.

2. Main results

We shall keep the same notation as in Section 1 and understand $\sum_{k=1}^{0}$ to be 0 as convention, and set further $\mathbb{I}\{\cdot\}$ the indicator function. Theorem 2.1 concerns tail asymptotics of extremes of X on Pickands grids $\mathfrak{R}(a_i q)$ with $a_1 = a_2/(m + l/n), a_2 > 0, m, n, l \in \mathbb{N}, m \geq 1, 1 \leq l < n\, (l = 0 \text{ if } n = 1)$. Denote