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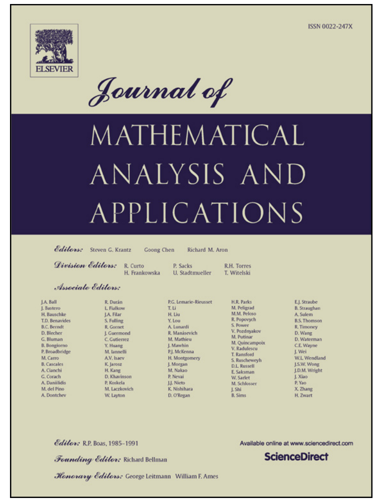

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A note on perturbations of Fusion Frames

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Abstract

In this work, we consider some relationships between a closed range operator T and a fusion frame $\mathcal{W} = (W_i, w_i)_{i \in I}$ for a Hilbert space $\mathcal{H}$ that provides that the sequence $(\overline{T(W_i)}, v_i)_{i \in I}$ is a fusion frame sequence for $\mathcal{H}$, if we consider a suitable family of weights $\{v_i\}_{i \in I}$. This (sufficient) condition generalizes some previous work in the subject.

Keywords: Fusion frames, Linear perturbation of Fusion Frames, Angle between subspaces
2000 MSC: 42C15, 15A60

1. Introduction

Let $\mathcal{H}$ be a separable Hilbert space. Let $\{W_i\}_{i \in I}$ be a family of closed subspaces of $\mathcal{H}$ and $\{w_i\}_{i \in I}$ be a family of positive weights. The pair $\mathcal{W} = (W_i, w_i)_{i \in I}$ is a **fusion frame** for $\mathcal{H}$ if there exists $A, B > 0$ which satisfy that

$$A\|f\|^2 \leq \sum_{i \in I} w_i^2 \|P_{W_i} f\|^2 \leq B\|f\|^2 \text{ for every } f \in \mathcal{H} \tag{1}$$

where by P_W we denote the orthogonal projection onto W. We say that $\mathcal{W} = (W_i, w_i)_{i \in I}$ is a **fusion frame sequence** for $\mathcal{H}$ if it is a fusion frame for a closed proper subspace of $\mathcal{H}$.

Fusion frames were introduced by P. Casazza and G. Kutyniok in [3], under the name of *frame of subspaces*. They were motivated by the problem of finding a way to join together local frames in order to get global frames. These authors were also studying methods to decompose a frame into a family of frame sequences. Both problems lead them to define the concept of frame of subspaces, which clearly generalizes classical vector frames. Namely, given a frame $\{f_i\}_{i \in I}$ for $\mathcal{H}$, then $(\text{span}\{f_i\}, \|f_i\|)_{i \in I}$ constitutes a fusion frame (of one-dimensional subspaces) for $\mathcal{H}$.

During the last decade, fusion frame theory has been a fast-growing area of research, driven by several applications such as sensor networks, neurology, coding theory, among others.

As well as for vector frames, there are some bounded operators associated to a fusion frame. First, we set the Hilbert space $\mathcal{K}_\mathcal{W} := \bigoplus_{i \in I} W_i$ (endowed with the ℓ^2 norm) and we define the **synthesis operator** $T_\mathcal{W} : \mathcal{K}_\mathcal{W} \to \mathcal{H}$ and the **analysis operator** $T_\mathcal{W}^* : \mathcal{H} \to \mathcal{K}_\mathcal{W}$ of $(W_i, w_i)_{i \in I}$ given by:

$$T_\mathcal{W}(g) = \sum_{i \in I} w_i \, g_i \text{ and } T_\mathcal{W}^*(f) = \{w_i \, P_{W_i} f\}_{i \in I}.$$

It is clear that some properties and problems in classical vector frames need to be treated in a completely different way if we are in the context of fusion frames. For example, problems of design of frames, such as the existence and construction of finite frames with prescribed norms need to be attacked with different tools when they are posed for fusion frames (see [2, 4, 13]).

Also, the equivalence between frames and epimorphisms useful in abstract frame theory, is not longer true for fusion frames (see for example [14]). Related to this, there is another interesting issue to study in

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