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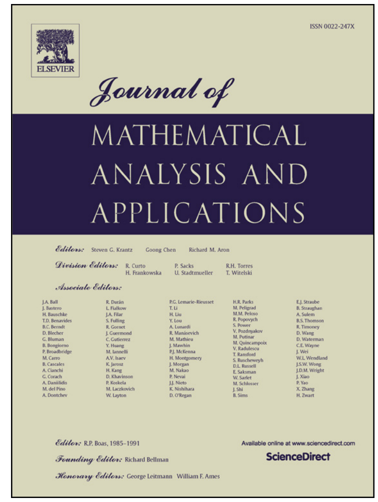

Central limit theorems for bounded random variables under belief measures

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Central limit theorems for bounded random variables under belief measures

Xiaomin Shi*

Abstract. Recently a new type of central limit theorem for belief functions was given in Epstein et al. [9]. In this paper, we generalize the central limit theorem in Epstein et al. [9] to accommodate general bounded random variables. These results are natural extension of the classical central limit theory for additive probability measures.

Key words. central limit theorem, belief measure, non-additive measure

1 Introduction

Recently, a central limit theorem (CLT for short) for belief functions was given in Epstein et al. [9] (Theorem 3.1) to construct suitably robust confidence regions for incomplete models. We state their CLT for the readers' convenience:

Theorem 1.1 *Let $\Lambda_{\theta_n} \to \Lambda \in \mathbb{R}^{J\cdot J}$ and $c_n \to c \in \mathbb{R}^J$. Then*

$$\nu_{\theta_n}^{\infty}(\cap_{j=1}^{J}\{s^{\infty} : \sqrt{n}[\nu_{\theta_n}(A_j) - \Psi_n(s^{\infty})(A_j)] \leq c_{nj}\}) \to \mathbf{N}_J(c;\Lambda). \tag{1.1}$$

In the theorem above, structure parameter θ and $J = 1, 2......$ be fixed a prior. S is a finite state space, and $A_1, ..., A_J$ are J subsets of S. $\Psi_n(s^{\infty})(A_j) = \frac{1}{n}\sum_{i=1}^{n} I_{\{s_i \in A_j\}}$, $j = 1, ..., J$ are the empirical frequency measure of A_j in the first n experiments along the sample $s^{\infty} = (s_1, s_2, ...)$.

$$cov_{\theta}(A_i, A_j) = \nu_{\theta}(A_i \cap A_j) - \nu_{\theta}(A_i)\nu_{\theta}(A_j). \tag{1.2}$$

Λ_{θ} is the $J \times J$ symmetric and positive semidefinite matrix $(cov_{\theta}(A_i, A_j))$ and ν_{θ} is a belief function on S.

$$\mathbf{N}_J(c;\Lambda) = P(\xi \leq c),$$

where ξ is a J-dimensional normal random variable with zero mean and covariance matrix Λ and relation $\xi \leq c$ is in the vector sense.

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