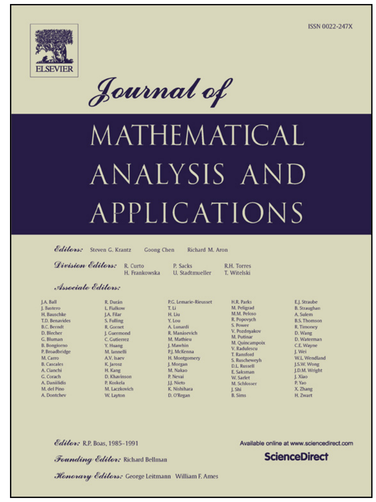

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LOCALISATION PROPERTY OF BATTLE–LEMARIÉ WAVELETS' SUMS

[1]

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Abstract: Explicit formulae are given for a type of Battle–Lemarié scaling functions and related wavelets. Compactly supported sums of their translations are established and applied to alternative norm characterization of sequence spaces isometrically isomorphic to Nikolskii–Besov spaces on $\mathbb{R}$.

1. Introduction

Battle–Lemarié scaling functions are polynomial splines with simple knots at $\mathbb{Z}$ obtained by orthogonalisation process of the B–splines. For $n \in \mathbb{N}$ the $n-$th order B–spline is defined recursively by

$$B_n(x) := (B_{n-1} * B_0)(x) = \int_0^1 B_{n-1}(x-t)\,dt = \frac{x}{n} B_{n-1}(x) + \frac{n+1-x}{n} B_{n-1}(x-1)$$

with $B_0 = \chi_{[0,1)}$. It is known that B_n generates multiresolution analysis of $L^2(\mathbb{R})$ [3]. Moreover [3],

a) $\operatorname{supp} B_n = [0, n+1]$, $n \in \mathbb{N} \cup \{0\}$ and $B_n(x) > 0$ for all $x \in (0, n+1)$, and $B_n \in C^{n-1}$ for $n \geq 1$;
b) the restriction of B_n to each $[m, m+1]$, $m = 0, \ldots, n$, is a polynomial of degree n;
c) the function $B_n(x)$ is symmetrical about $x = (n+1)/2$, that is $B_n(\frac{n+1}{2} - x) = B_n(\frac{n+1}{2} + x)$.

Battle–Lemarié scaling functions and related wavelets play an important role in approximation theory, numerical analysis (see e.g. [13]), image, data and signal processing involving analysis of biological sequences and molecular biology–related signals, etc. [12]. On the strength of the differentiation property

$$B_n'(x) = B_{n-1}(x) - B_{n-1}(x-1) \qquad \text{for a.a. } x \in \mathbb{R}, \qquad n \in \mathbb{N}, \tag{1}$$

this function class has appeared to be an effective tool for solving problems related to the theory of integration and differentiation operators in function spaces [14]. There is a number of papers devoted to Battle–Lemarié scaling functions and wavelets. Most of them deal with their implicit or approximate expressions. An idea of how to find explicit formulae for this function class was given in works by I.Ya. Novikov and S.B. Stechkin ([17, § 7] and [18, § 15], see also [15] and [16]). The problem, we dealt in [14], concerned the operators' compactness and approximation properties. The solution method required, first of all, explicit formulae for the chosen wavelet system. The other important points were in finding their proper transformations and sums in order to localise non–compactly supported scaling functions and wavelets of this type and, making use of (1), connect their components (splines) with splines of lower or higher orders. All these questions were covered in [14, § 2.2.2, Lemma 5, Proposition 7, Corollary 8] for the scaling function and wavelets of the first order only. The results of the present work make us able to continue the study of compactness and approximation properties of integration operators in

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