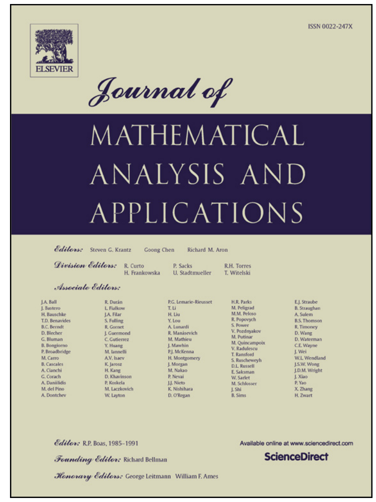
Accepted Manuscript



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PII: S0022-247X(18)30018-0
DOI: https://doi.org/10.1016/j.jmaa.2018.01.011
Reference: YJMAA 21944

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 1 December 2017

An Initial Boundary Value Problem for Screw Pinches in Plasma Physics with Temperature Dependent Viscosity Coefficients*

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Abstract. This paper is concerned with an initial-boundary value problem for screw pinches arisen from plasma physics. A global-in-time, classical solution to this physically very important problem, is shown to exist uniquely and converge exponentially to the constant state as the time tends to infinity under certain assumptions on the initial data and the adiabatic exponent γ. The main difficulties in the proof lie in the temperature dependent magnetic resistive coefficients induced by the magnetic field. The initial data can be large if γ is sufficiently close to 1.

Keywords. Screw pinch; plasma physics; magnetohydrodynamics (MHD); global classical solution; long time behavior

AMS Subject Classifications (2000). 35B45, 35L65, 35Q60, 76N10

1 Introduction

In this paper, the following Magnetohydrodynamics (MHD) system is concerned,

$$
\begin{cases}
\rho_t + \operatorname{div}(\rho \boldsymbol{u}) = 0, \qquad \boldsymbol{x} \in \mathbb{R}^3, \quad t \in \mathbb{R}^+, \\
(\rho \boldsymbol{u})_t + \operatorname{div}(\rho \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla P = (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \operatorname{div}\mathbb{S}, \\
\boldsymbol{B} - \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) = -\nabla \times (\sigma \nabla \times \boldsymbol{B}), \quad \operatorname{div}\boldsymbol{B} = 0, \\
\left(\rho e + \frac{1}{2}\rho|\boldsymbol{u}|^2 + \frac{1}{2}|\boldsymbol{B}|^2\right)_t + \operatorname{div}\left(\left(\rho e + \frac{1}{2}\rho|\boldsymbol{u}|^2 + P\right)\boldsymbol{u} - \kappa\nabla\theta\right) \\
\qquad = \operatorname{div}((\boldsymbol{u} \times \boldsymbol{B}) \times \boldsymbol{B} + \sigma \boldsymbol{B} \times (\nabla \times \boldsymbol{B}) + \mathbb{S}\boldsymbol{u}).
\end{cases} \tag{1.1}
$$

As it is well known, the motion of a conducting fluid (plasma) in an electromagnetic field is governed by the equations of MHD, which is a coupled system of the induction equation of the magnetic field and the Navier-Stokes equations of fluid dynamics (see also [1, 5, 6, 7, 8, 9, 14]). Here, ρ is the density, $\boldsymbol{u} \in \mathbb{R}^3$ the velocity, $\boldsymbol{B} \in \mathbb{R}^3$ the magnetic field, and θ the temperature; the pressure P and the internal energy e are related with the density and temperature of the flow by the equations of state:

$$
P = P(\rho, \theta) = R\rho\theta = A\rho^\gamma \exp\left(\frac{\gamma - 1}{R}s\right), \quad \text{and} \quad e = e(\rho, \theta) = c_v\theta = \frac{R\theta}{\gamma - 1}, \tag{1.2}
$$

*Supported by NNSFC (Grant No. 11271306) and the Natural Science Foundation of Fujian Province of China (Grant No.2015J01023)

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