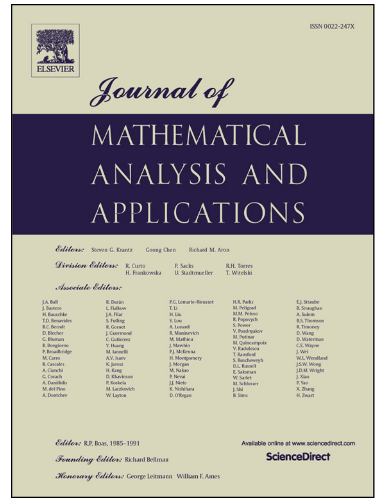

A regularity criterion for the 3D incompressible Magneto-hydrodynamics equations

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PII: S0022-247X(17)31095-8
DOI: https://doi.org/10.1016/j.jmaa.2017.12.017
Reference: YJMAA 21878

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 28 November 2016

Please cite this article in press as: F. Xu, A regularity criterion for the 3D incompressible Magneto-hydrodynamics equations, *J. Math. Anal. Appl.* (2018), https://doi.org/10.1016/j.jmaa.2017.12.017

A regularity criterion for the 3D incompressible Magneto-hydrodynamics equations *

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Abstract. The paper is dedicated to study of the regularity criterion for weak solutions to the 3D incompressible MHD equations. Employing the Littlewood-Paley decomposition, we show that if $\tilde{\nabla}\tilde{u} = (\partial_1\tilde{u}, \partial_2\tilde{u}) \in L^{s_1}([0,T); \dot{B}^0_{r_1,\frac{2r_1}{3}}(\mathbb{R}^3))$, $\frac{2}{s_1} + \frac{3}{r_1} = 2$, $\frac{3}{2} < r_1 \leq \infty$ and $\tilde{\nabla}\tilde{b} = (\partial_1\tilde{b}, \partial_2\tilde{b}) \in L^{s_2}([0,T); \dot{B}^0_{r_2,\frac{2r_2}{3}}(\mathbb{R}^3))$, $\frac{2}{s_2} + \frac{3}{r_2} = 2$, $\frac{3}{2} < r_2 \leq \infty$, then the solutions to the MHD actually is smooth on $(0,T)$.

Key words. MHD equations, Regularity criterion, Littlewood-Paley decomposition

AMS subject classifications. 76W05 35B65

1 Introduction

We consider the 3D incompressible magneto-hydrodynamics (MHD) equations

$$
\text{(MHD)} \begin{cases} \dfrac{\partial u}{\partial t} - \nu\Delta u + u\cdot\nabla u = -\nabla p - \dfrac{1}{2}\nabla|b|^2 + b\cdot\nabla b, \\ \dfrac{\partial b}{\partial t} - \eta\Delta b + u\cdot\nabla b = b\cdot\nabla u, \\ \nabla\cdot u = \nabla\cdot b = 0, \\ u(0,x) = u_0(x), \quad b(0,x) = b_0(x). \end{cases} \tag{1.1}
$$

Here u, b describe the flow velocity vector and the magnetic field vector respectively, p is a scalar pressure, $\nu > 0$ is the kinematic viscosity and $\eta > 0$ is the magnetic diffusivity, while u_0 and b_0 are the given initial velocity and initial magnetic field respectively, with $\nabla\cdot u_0 = \nabla\cdot b_0 = 0$. If $\nu = \eta = 0$, (1.1) is called the ideal MHD equations. Using the standard energy method, it can be easily proved that for given initial data $(u_0, b_0) \in H^s(\mathbb{R}^3)$ with $s > \frac{1}{2}$, there exists a

*Research supported by the National Natural Science Foundation of China (11501332,11171034,11371221), the Natural Science Foundation of Shandong Province (ZR2015AL007), China Postdoctoral Science Foundation funded project (2014M561893), Postdoctoral innovation fund of Shandong Province, the Open Fund of State Key Laboratory of Simulation and Regulation of Water Cycle in River Basin, China Institute of Water Resources and Hydropower Research Fund(IWHR-SKL-201407), and the Specialized Research Foundation for the Doctoral Program of Higher Education of China (20123705110001), and Young Scholars Research Fund of Shandong University of Technology.

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