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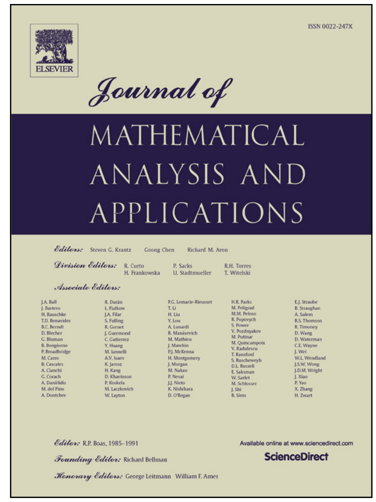

Asymptotically Non-expansive Actions of Strongly Amenable Semigroups and Fixed Points

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Abstract

In this paper, we deal with some fixed point properties for a semi-topological semigroup $\mathcal{S}$ acting on a compact convex subset K of a Banach space. We first focus on the space $LMC(\mathcal{S})$ of left multiplicatively continuous functions on S and its strong left amenability; the existence of a compact left ideal group in the LMC-compactification of $\mathcal{S}$. We then study the relation between left amenability and strong left amenability of $LMC(\mathcal{S})$ with a common fixed point property for non-expansive and asymptotically non-expansive actions of $\mathcal{S}$. Our results improve a result of T. Mitchell in 1970, and answer an open problem of A. T.-M. Lau in 2010 for the class of strongly left amenable semi-topological semigroups. [1]

1 Introduction

Let $\mathcal{S}$ be a *semi-topological semigroup*; i.e., $\mathcal{S}$ is a semigroup with a Hausdorff topology such that for each $t \in \mathcal{S}$, the mappings $s \mapsto st$ and $s \mapsto ts$ from $\mathcal{S}$ into $\mathcal{S}$ are continuous. A semi-topological semigroup $\mathcal{S}$ is called *right (left) reversible* if any two closed left (right) ideals of $\mathcal{S}$ have non-void intersection. An *action* of $\mathcal{S}$ on a subset K of a Hausdorff topological space is a mapping of $\mathcal{S} \times K$ into K denoted by $(s, x) \mapsto s \cdot x$ such that

$$(st) \cdot x = s \cdot (t \cdot x)$$

for all $s, t \in \mathcal{S}$ and $x \in K$. A *common fixed point* for $\mathcal{S}$ in K is a point x_0 in K such that $s \cdot x_0 = x_0$ for all $s \in \mathcal{S}$. The action of $\mathcal{S}$ on K is called *separately continuous* if it is continuous in each variable when the other is kept fixed. When K is a subset of a Banach space, the action is called *asymptotically non-expansive* if for each $x, y \in K$, there is a left ideal $\mathcal{J} \subseteq \mathcal{S}$ such that

$$\|s \cdot x - s \cdot y\| \leq \|x - y\|$$

for all $s \in \mathcal{J}$; see [15] for details. Particular examples of asymptotically non-expansive actions are *non-expansive actions*; that is, actions for which the mappings $x \mapsto s \cdot x$ from K into K are non-expansive for all $s \in \mathcal{S}$; recall that a mapping $T : K \to K$ is called *non-expansive* if

$$\|Tx - Ty\| \leq \|x - y\|$$

for all $x, y \in K$. In 1963, DeMarr was proved that

[1] 2010 *Mathematics Subject Classification*: Primary 20M30, 22A20, 43A07, 47H10. Secondary 43A15, 47H20.

Key words: Asymptotically non-expansive action, Banach space, left ideal groups, left multiplicatively continuous functions, fixed point property, right reversible semigroup, semi-topological semigroup, strong amenability.