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Global behavior for the classical Nicholson–Bailey model

William T. Jamieson [a,*], Jenna Reis [b]

[a] Department of Mathematics, Southern New Hampshire University, Manchester, NH 03106, United States
[b] Department of Mathematics, Fitchburg State University, Fitchburg, MA 01420, United States

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ABSTRACT

This article investigates the global asymptotic behavior of the classical Nicholson–Bailey model [6] for $\lambda > 1$. In particular, it is shown that the Nicholson–Bailey model has no periodic solutions in the first quadrant other than the fixed point $(\bar{x}, \bar{y})$ and that all non-trivial solutions in the first quadrant are unbounded.

1. Introduction

A general host parasitoid model has the form

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} c\, y_n(1 - g(x_n, y_n)) \\ \lambda\, y_n g(x_n, y_n) \end{pmatrix}, \quad \text{where } c, \lambda > 0 \text{ and } x_0, y_0 > 0. \tag{1}$$

The variables x_n and y_n represent the populations of the parasitoid and the host at time n, respectively. The parameter $\lambda > 0$ represents the intrinsic growth rate of the host, the parameter $c > 0$ represents the number of viable eggs laid by a single parasitoid, and $g(x, y)$ represents the probability that a host escapes being infested by a parasitoid. In 1935, Nicholson and Bailey [6] made the assumption that the probability of a host coming into contact with a parasitoid is modeled by a Poisson distribution in the number of parasitoids, and that a host becomes infested after the first contact with a parasitoid; that is $g(x, y) = e^{-ax}$, where the constant $a > 0$ depends on the parasitoid's ability to locate hosts. After scaling $x \mapsto a\,x$ and $y \mapsto c\,a\,y$, we have the Nicholson–Bailey host parasitoid model

$$F\begin{pmatrix} x_n \\ y_n \end{pmatrix} := \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} y_n(1 - e^{-x_n}) \\ \lambda\, y_n e^{-x_n} \end{pmatrix}, \quad \text{where } \lambda > 0 \text{ and } x_0, y_0 > 0. \tag{NB}$$

* Corresponding author.
E-mail addresses: w.jamieson@snhu.edu (W.T. Jamieson), jreis5@fitchburgstate.edu (J. Reis).

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Let

$$\mathcal{Q} = \{(x, y) \in \mathbb{R}^2 : x, y > 0\}.$$

The fixed points of $F(x, y)$ in $\mathbb{R}^2$ are $(0, 0)$ and

$$(\bar{x}, \bar{y}) = \left(\ln\lambda, \frac{\lambda}{\lambda - 1}\ln\lambda\right).$$

The fixed point $(\bar{x}, \bar{y})$ lies in $\mathcal{Q}$ if $\lambda > 1$. If $0 < \lambda < 1$ all solutions in $\mathcal{Q}$ converge to the fixed point $(0, 0)$, and in the non-hyperbolic case $\lambda = 1$, all solutions lie on the level curves of the function $z = x + y - \ln y$; see [4]. If $\lambda > 1$, then $(\bar{x}, \bar{y})$ is an unstable focus. Hsu et al. proved in this case that all solutions are oscillatory; see Theorem 4.4 in [4]:

Theorem A *(Hsu et al.). Let $\lambda > 1$ and $(x, y) \in \mathcal{Q}$ be any point different from $(\bar{x}, \bar{y})$. Then the bisequences $\{x_n\}_{n=-\infty}^{\infty}$ and $\{y_n\}_{n=-\infty}^{\infty}$ generated by iterating $F(x, y)$ are strictly oscillatory around $\bar{x}$ and $\bar{y}$ respectively. Moreover, in polar coordinates (r, θ) centered at $(\bar{x}, \bar{y})$, for the bisequence $\{(x_n, y_n)\}_{n=-\infty}^{\infty}$ the corresponding bisequence $\{\theta_n\}_{n=-\infty}^{\infty}$ of polar angles is strictly decreasing and satisfies*

$$\lim_{n\to\infty} \theta_n = -\infty \quad \textit{and} \quad \lim_{n\to-\infty} \theta_n = \infty.$$

Moreover the authors also proved the non-existence of periodic orbits for $\lambda > 1$ sufficiently close to 1 (see [4] Corollary 4.6):

Theorem B *(Hsu et al.). For any positive integer N, there exists $\lambda_0 > 1$ such that for any $1 < \lambda < \lambda_0$, $F_\lambda(x, y)$ has no periodic points in $\mathcal{Q}$ of period less than N except the fixed point $(\bar{x}, \bar{y})$.*

In this article, Theorem B will be improved by showing that for all $\lambda > 1$, $F(x, y)$ has no periodic solutions other than the fixed point $(\bar{x}, \bar{y})$. Further, it will be proven that all non-trivial solutions in the first quadrant of system (NB) are unbounded. These results verify the numerical observation [2] that when $\lambda > 1$, both the host and parasitoid populations are unbounded and oscillate with increasing amplitude.

2. Preliminary lemmas

The proof of the main theorem will rely on the following function:

$$V(x, y) = \frac{\ln\lambda}{\lambda - 1}(x - \ln\lambda \ln x) + \frac{1}{\lambda}\left(y - \left(\frac{\lambda\ln\lambda}{\lambda - 1}\right)\ln y\right).$$

Define

$$L(x, y) = V(x, y) - V(\bar{x}, \bar{y})$$

and

$$\Delta L(x, y) = L(F(x, y)) - L(x, y). \tag{2}$$

The functions $V(x, y)$, $L(x, y)$, and $\Delta L(x, y)$ depend on the value of λ, but this dependency will be suppressed in the notation. Level curves for the function $L(x, y)$ are depicted in Fig. 1. It should be noted that $V(x, y)$ has the form of solutions to the Volterra–Lotka predator–prey equations