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Stability and bifurcation analysis of micro-electromechanical nonlinear coupling system with delay

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In this paper, we study dynamics in delayed micro-electromechanical nonlinear coupling system, with particular attention focused on Hopf and Hopf-pitchfork bifurcations. Based on the distribution of eigenvalues, we prove that a sequence of Hopf and Hopf-pitchfork bifurcations occur at the trivial equilibrium as the delay increases and obtain the critical values of two types of bifurcations. Next, by applying the multiple time scales method, the normal forms near the Hopf and Hopf-pitchfork bifurcations critical points are derived. Finally, bifurcation analysis and numerical simulations are presented to demonstrate the application of the theoretical results. We show the regions near above bifurcation critical points in which the micro-electromechanical nonlinear coupling system exists stable fixed point or stable periodic solution. Detailed numerical analysis using MATLAB extends the local bifurcation analysis to a global picture, and stable windows are observed as we change control parameters. Namely, the stable fixed point and stable periodic solution can exist in large regions of unfolding parameters as the unfolding parameters increase away from the critical value.

1. Introduction

Micro-electromechanical systems (MEMS, also written as micro-electro-mechanical, micro-electro-mechanical or microelectronic and microelectromechanical systems and the related micromechatronics) are the technology of microscopic devices, particularly those with moving parts. They merge at the nano-scale into nanoelectromechanical systems (NEMS) and nanotechnology. MEMS are separate and distinct from the hypothetical vision of molecular nanotechnology or molecular electronics. They usually consist of a central unit that processes data (the microprocessor) and several components that interact with the surroundings such as microsensors. At these size scales, the standard constructs of classical physics are not always sufficient. Because of the large surface area to volume ratio of MEMS, surface effects such as electrostatics and wetting dominate over volume effects such as inertia or thermal mass. MEMS become practical once

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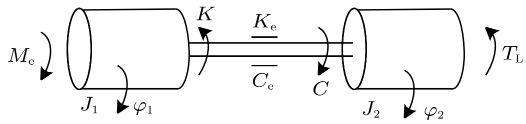

Fig. 1. Dynamic model of the electromechanical coupling system driven by an AC motor.

they could be fabricated using modified semiconductor device fabrication technologies, normally used to make electronics. These include molding and plating, wet etching (KOH, TMAH) and dry etching (RIE and DRIE), electro discharge machining (EDM), and other technologies capable of manufacturing small devices. An early example of MEMS device is the resonistor, that is, an electromechanical monolithic resonator [7,16].

The MEMS represent a very important class of systems having applications in all fields. Because of the nonlinearity and the complexity, modeling and dynamics of MEMS have strongly attracted scholar's attention. MEMS often involve the nonlinear coupling of electrostatic and mechanical physical fields in engineering, so the dynamic characters are complicated. The transmission system driven by an alternating current (AC) motor as a core part of rotating machinery plays an irreplaceable role in electricity, energy, transportation, metallurgy, and national defense fields. The torsional vibration of the rotating machinery will affect the normal work and cause the equipment to be damaged [9,10]. The rolling mill main drive system driven by an AC motor can be abstracted into a two-mass torsional vibration model, as shown in Fig. 1.

By considering the energy in the air-gap field of the AC motor, the dynamical equation of the electromechanical coupling transmission system, associated with a two masses relative rotation system driven by an AC motor, is deduced as follows [9,10]:

$$\begin{cases} J_1\ddot{\varphi}_1 + K(\varphi_1 - \varphi_2) + C(\dot{\varphi}_1 - \dot{\varphi}_2) + C_e\dot{\varphi}_1 - K_e\varphi_1 = M_e, \\ J_2\ddot{\varphi}_2 - K(\varphi_1 - \varphi_2) - C(\dot{\varphi}_1 - \dot{\varphi}_2) = M_d, \end{cases} \tag{1}$$

where $J_i > 0$ $(i = 1, 2)$ is the moment of inertia; φ_i $(i = 1, 2)$ and $\dot{\varphi}_i$ $(i = 1, 2)$ are angle of rotation and rotational speed, respectively; $K > 0$ is the torsional stiffness of drive shaft; $C > 0$ is the shafting damping coefficient; $C_e > 0$ is the electromagnetic damping coefficient (generated by the rotor damper bar); M_e is the electromagnetic torque, and M_d is the load torque. The detailed deducing can be referred to Refs. [9,10]. Equation (1) is the nonlinear dynamic equation of two masses relative rotation system considering both electrical parameters and mechanical parameters, which is the basis of dynamic behavior analysis of electromechanical coupling relative rotation system.

Time delay, as a kind of basic nature of the physical phenomenon, exists widely in industrial systems. It has a great influence on the analysis and the control of a system, such as leading to the instability of the system and the control law failure. However, reasonably introducing and processing the time delay can improve the performance of a system. It is well known that delay differential equations (DDE) may exhibit higher codimension singularities more frequently than that in ordinary differential equations (ODE) [1,2,8, 13–15,17,20].

In order to control the dynamic behaviors of the electromechanical coupling system, the nonlinear time delay state feedback term just like $\tilde{k}_1\varphi_2(t-\tau) + \tilde{k}_2\varphi_2^3(t-\tau)$ is introduced, that is, $M_d = \tilde{k}_1\varphi_2(t-\tau) + \tilde{k}_2\varphi_2^3(t-\tau)$. In the case of no eccentricity, $M_e = \partial W_m/\partial\varphi_1 = \mu_0 + \mu_1\varphi_1 + \mu_2\varphi_1^2 + \mu_3\varphi_1^3$, where W_m is air-gap magnetic field energy between the stator and rotor,

$$\mu_0 = \tilde{\mu}_0 \sin(\psi_1 + \psi_2),\ \ \mu_1 = \tilde{\mu}_1 \cos(\psi_1 + \psi_2),$$
$$\mu_2 = \tilde{\mu}_2 \sin(\psi_1 + \psi_2),\ \ \mu_3 = \tilde{\mu}_3 \cos(\psi_1 + \psi_2),$$