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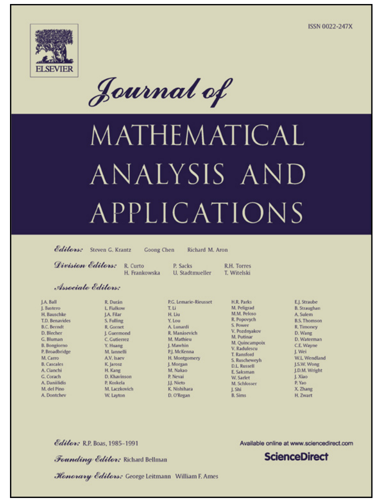

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On the existence of small energy solutions for a sublinear Neumann problem

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Abstract: In this paper, we are concerned with the sublinear problem

$$\begin{cases} -\Delta u = |u|^{p-2}u & \text{in } \Omega, \\ u_\nu = 0 & \text{on } \partial\Omega, \end{cases} \tag{0.1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, and $1 \le p < 2$. For $p = 1$, the nonlinearity $|u|^{p-2}u$ will be identified by $\operatorname{sgn}(u)$. In contrast to previous work on the Dirichlet problem, some difficulties arise due to the fact that the associated energy functional is not bounded from below. Complementing recent work by Parini and Weth in [15] on least energy solutions, we prove that (0.1) has infinitely many solutions with small negative energy.

MSC: Primary 35J25; Secondary 35J20, 49J52

Keywords: Sublinear Neumann problems; Nonsmooth analysis; Subdifferentials; Small energy solutions

1 Introduction

The present paper is devoted to the existence of small energy solutions for the following sublinear Neumann boundary value problem

$$\begin{cases} -\Delta u = |u|^{p-2}u & \text{in } \Omega, \\ u_\nu = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N (N \ge 1)$ is a bounded domain with Lipschitz boundary, and $1 \le p < 2$. Here u_ν is the outer normal derivative of u at the boundary $\partial\Omega$, and the nonlinear term $|u|^{p-2}u$ will be identified by $\operatorname{sgn}(u)$ in case $p = 1$ in the following.

For the case $p > 1$, problem (1.1) arises e.g. in the study of quasistationary solutions of the form $w(t, x) = u(x)T(t)$ for the (sign changing) porous medium equation, see e.g. [15, pp. 708]. For a more detailed discussion of the (sign changing) porous medium equation, see [18] and the references therein.

In the case $p = 1$, (1.1) is contained in the class of general elliptic boundary value problems with discontinuous nonlinearities. Such problems are solved e.g. by equilibria of reaction diffusion equations with discontinuous reaction terms, see e.g. [2, 5, 14, 16].

Let us now consider the energy functional

$$\varphi : H^1(\Omega) \to \mathbb{R}, \qquad \varphi(u) = \frac{1}{2}\int_\Omega |\nabla u|^2\, dx - \frac{1}{p}\int_\Omega |u|^p\, dx. \tag{1.2}$$

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