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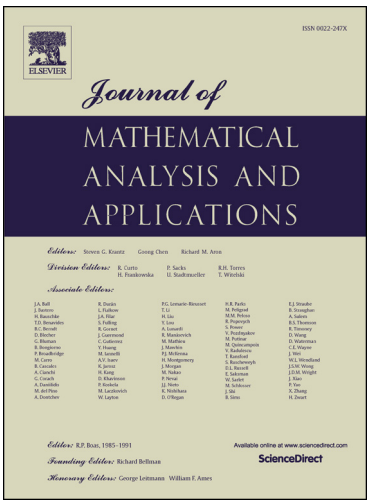

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Expansivity and Shadowing in Linear Dynamics

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Abstract

In the early 1970's Eisenberg and Hedlund investigated relationships between expansivity and spectrum of operators on Banach spaces. In this paper we establish relationships between notions of expansivity and hypercyclicity, supercyclicity, Li-Yorke chaos and shadowing. In the case that the Banach space is c_0 or ℓ_p $(1 \leq p < \infty)$, we give complete characterizations of weighted shifts which satisfy various notions of expansivity. We also establish new relationships between notions of expansivity and spectrum. Moreover, we study various notions of shadowing for operators on Banach spaces. In particular, we solve a basic problem in linear dynamics by proving the existence of nonhyperbolic invertible operators with the shadowing property. This contrasts with the expected results for nonlinear dynamics on compact manifolds, illuminating the richness of dynamics of infinite dimensional linear operators.[1]

1 Introduction

The study of the dynamics of continuous linear operators on infinite dimensional Banach (or Fréchet) spaces has witnessed a great development during the last three decades and many links between this area and other areas of mathematics, such as ergodic theory, number theory and geometry of Banach spaces, have been established. We refer the reader to the books [3, 18] and to the more recent papers [4, 7, 8, 9, 17], where many additional references can be found. On the other hand, the notions of expansivity and shadowing play important roles in many branches of the area of dynamical systems, including topological dynamics, differentiable dynamics and ergodic theory; see [1, 23, 24, 30], for instance. Our goal in this paper is to investigate the notions of expansivity and shadowing in the context of linear dynamics, thereby complementing previous works by various authors. In particular, we give a class of examples of operators exhibiting the shadowing property which are not hyperbolic, however they are chaotic. Such examples show the richness of linear dynamics and its difference from finite dimensional nonlinear dynamics, yielding counterintuitive results to the corresponding ones from finite dimensional smooth dynamics.

Let us now describe the organization of the article.

In Section 2 we fix the notations and recall the definitions and a few results which will be important in our work.

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