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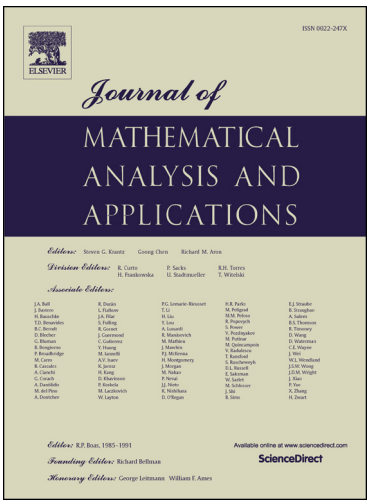

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S. Bautista, C.A. Morales, H. Villavicencio

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DESCRIPTIVE SET THEORY FOR EXPANSIVE SYSTEMS

S. BAUTISTA, C.A. MORALES, H. VILLAVICENCIO

Abstract. Kato [5] and Artigue [3] merged the theory of expansive systems [10] and foliations [17] with the continuum theory [14]. Here we merge the expansive systems but with the descriptive set theory [6] instead. More precisely, we define *meagre-expansivity* for both homeomorphisms and measures by requiring the *interior* of the dynamical balls up to some prefixed radio to be either empty or with zero measure respectively. We first prove that every cw-expansive homeomorphism of a locally connected metric space without isolated points is meagre-expansive (but not conversely). Second that a homeomorphism of a metric space is meagre-expansive if and only if every Borel probability measure is meagre-expansive. Next that the space of meagre-expansive measures of a homeomorphism of a compact metric space X is an F_σ subset of the space of Borel probability measures equipped with the weak* topology. In the sequel we prove that every homeomorphism with a meagre-expansive measure of a compact metric space has an *invariant* meagre-expansive measure. Also that the set of periodic points of every meagre-expansive homeomorphism of a compact metric space has empty interior. In the circle or the interval we prove that there are no meagre-expansive homeomorphisms of the circle or the interval. Moreover, the meagre-expansive measures of an interval homeomorphism or a circle homeomorphism with rational rotation number are precisely the finite convex combinations of Dirac measures supported on isolated periodic points. A circle homeomorphism with irrational rotation number has a meagre-expansive measure if and only if it is a Denjoy map. In such a case the meagre-expansive measures are precisely those measures supported on the unique minimal set of the map. To obtain some of our results we will consider a measurable version of the classical Baire Category.

1. Introduction

Trajectories in dynamics have two antagonistic behaviors: to stay either separated or close each other respectively. The former behavior, the leitmotiv of this work, first distinguishes by Utz [19] is nowadays known as *expansivity*. Several outgoings dealing with generalizations of this definition have been carried out along these decates or so. These include the *cw-expansivity* [5], the *N-expansivity* [12], the *cwN-expansivity* [3] which form together the so-called levels of expansivity [11]. The corresponding levels for measures have been studied through the notions of *expansive measure* ([2], [13]), and the N, *cw* and *cwN-expansive measures* [8], [18].

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