Accepted Manuscript

Dual Truncated Toeplitz Operators

Xuanhao Ding, Yuanqi Sang

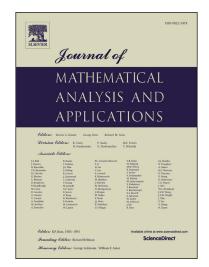
PII: S0022-247X(17)31110-1

DOI: https://doi.org/10.1016/j.jmaa.2017.12.032

Reference: YJMAA 21893

To appear in: Journal of Mathematical Analysis and Applications

Received date: 27 July 2017



Please cite this article in press as: X. Ding, Y. Sang, Dual Truncated Toeplitz Operators, *J. Math. Anal. Appl.* (2018), https://doi.org/10.1016/j.jmaa.2017.12.032

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

ACCEPTED MANUSCRIPT

DUAL TRUNCATED TOEPLITZ OPERATORS

XUANHAO DING AND YUANQI SANG*

ABSTRACT. Let u be a nonconstant inner function. In this paper, we study the dual truncated Toeplitz operators on the orthogonal complement of the model space K_u^2 . This is a new class of Toeplitz operators. We show that the product of two dual truncated Toeplitz operators $D_f D_g$ to be zero if and only if either f or g is zero. We give a necessary and sufficient condition for the product of two dual truncated Toeplitz operators to be a finite rank operator. Furthermore, a necessary and sufficient condition is found for the product of two dual truncated Toeplitz operators to be a dual truncated Toeplitz operator. The last two results are different from the classical Toeplitz operator theory.

1. Introduction and preliminaries

Let $\mathbb{D}=\{\xi\in\mathbb{C}:|\xi|<1\}$ be the unit disk in the complex plane \mathbb{C} and $\partial\mathbb{D}$ be its boundary. By H^2 is meant the standard Hardy space, the Hilbert space of holomorphic functions in \mathbb{D} having square-summable Taylor coefficients at the origin. As usual, H^2 will be identified with its space of boundary functions, the subspace of $L^2(\partial\mathbb{D})$ (of normalized Lebesgue measure m on $\partial\mathbb{D}$) consisting of functions whose Fourier coefficients with negative indices vanish.

Let P be the projection from $L^2(\partial \mathbb{D})$ to H^2 . For f in $L^2(\partial \mathbb{D})$, the Toeplitz operator T_f with symbol f is densely defined on H^2 by

$$T_f x = P(f x)$$
, for $x \in H^{\infty}$.

The dual Toeplitz operator S_f , on the orthogonal complement of H^2 would be densely defined as follows:

$$S_f y = (I - P)(fy), \text{ for } y \in [H^2]^{\perp} \cap L^{\infty}(\partial \mathbb{D}).$$

The Hankel operator H_f with symbol f is densely defined by

$$H_f x = (I - P)(fx), \text{ for } x \in H^{\infty},$$

and H_f^* is densely defined by

$$H_f^* y = P(\bar{f}y), \text{ for } y \in [H^2]^{\perp} \cap L^{\infty}(\partial \mathbb{D}).$$

Write M_f for the multiplication operator defined on L^2 by $M_f \varphi = f \varphi$. If M_f is expressed as an operator matrix with respect to the decomposition $L^2(\partial \mathbb{D}) = H^2 \oplus \overline{zH^2}$, the result is of the form

$$M_f = \begin{pmatrix} T_f & H_{\overline{f}}^* \\ H_f & S_f \end{pmatrix}.$$

Since $M_f M_g = M_{fg}$, we have

$$T_{fg} = T_f T_g + H_{\bar{f}}^* H_g; (1.1)$$

$$H_{fg} = H_f T_g + S_f H_g; (1.2)$$

$$S_{fg} = S_f S_g + H_f H_{\overline{g}}^*. \tag{1.3}$$

 $^{2010\ \}textit{Mathematics Subject Classification}.\ \text{Primary: 47B35; Secondary: 47B38}.$

Key words and phrases. Hardy space, Model space, Dual Truncated Toeplitz operator, Hankel operator.

The first author is supported by NSFC of China(11271388) and the Program for University Innovation Team of Chongqing (CXTDX201601026). The second author was supported by NSFC of China (11501059).

^{*}Corresponding author: yqisang@163.com.

Download English Version:

https://daneshyari.com/en/article/8900085

Download Persian Version:

https://daneshyari.com/article/8900085

<u>Daneshyari.com</u>