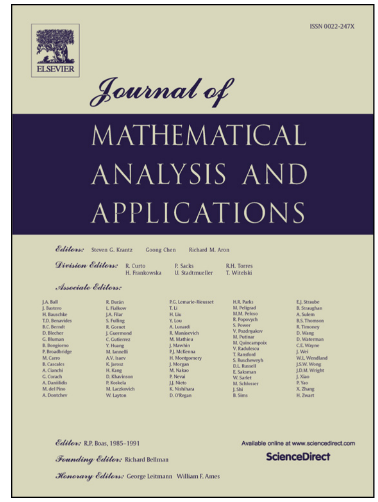

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DUAL TRUNCATED TOEPLITZ OPERATORS

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Abstract. Let u be a nonconstant inner function. In this paper, we study the dual truncated Toeplitz operators on the orthogonal complement of the model space K_u^2. This is a new class of Toeplitz operators. We show that the product of two dual truncated Toeplitz operators $D_f D_g$ to be zero if and only if either f or g is zero. We give a necessary and sufficient condition for the product of two dual truncated Toeplitz operators to be a finite rank operator. Furthermore, a necessary and sufficient condition is found for the product of two dual truncated Toeplitz operators to be a dual truncated Toeplitz operator. The last two results are different from the classical Toeplitz operator theory.

1. Introduction and preliminaries

Let $\mathbb{D} = \{\xi \in \mathbb{C} : |\xi| < 1\}$ be the unit disk in the complex plane $\mathbb{C}$ and $\partial\mathbb{D}$ be its boundary. By H^2 is meant the standard Hardy space, the Hilbert space of holomorphic functions in $\mathbb{D}$ having square-summable Taylor coefficients at the origin. As usual, H^2 will be identified with its space of boundary functions, the subspace of $L^2(\partial\mathbb{D})$(of normalized Lebesgue measure m on $\partial\mathbb{D}$) consisting of functions whose Fourier coefficients with negative indices vanish.

Let P be the projection from $L^2(\partial\mathbb{D})$ to H^2. For f in $L^2(\partial\mathbb{D})$, the Toeplitz operator T_f with symbol f is densely defined on H^2 by

$$T_f x = P(fx), \text{ for } x \in H^\infty.$$

The dual Toeplitz operator S_f, on the orthogonal complement of H^2 would be densely defined as follows:

$$S_f y = (I-P)(fy), \text{ for } y \in [H^2]^\perp \cap L^\infty(\partial\mathbb{D}).$$

The Hankel operator H_f with symbol f is densely defined by

$$H_f x = (I-P)(fx), \text{ for } x \in H^\infty,$$

and H_f^* is densely defined by

$$H_f^* y = P(\bar{f} y), \text{ for } y \in [H^2]^\perp \cap L^\infty(\partial\mathbb{D}).$$

Write M_f for the multiplication operator defined on L^2 by $M_f\varphi = f\varphi$. If M_f is expressed as an operator matrix with respect to the decomposition $L^2(\partial\mathbb{D}) = H^2 \oplus \overline{zH^2}$, the result is of the form

$$M_f = \begin{pmatrix} T_f & H_{\bar{f}}^* \\ H_f & S_f \end{pmatrix}.$$

Since $M_f M_g = M_{fg}$, we have

$$T_{fg} = T_f T_g + H_{\bar{f}}^* H_g; \tag{1.1}$$

$$H_{fg} = H_f T_g + S_f H_g; \tag{1.2}$$

$$S_{fg} = S_f S_g + H_f H_{\bar{g}}^*. \tag{1.3}$$

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