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Controllability aspects of the Korteweg–de Vries Burgers equation on unbounded domains

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This paper is dedicated to my newborn daughter, Amelia Gallego Gómez

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ABSTRACT

The aim of this work is to consider the controllability problem of the linear system associated to Korteweg–de Vries Burgers equation posed in the whole real line. We obtain a sort of exact controllability for solutions in $L^2_{loc}(\mathbb{R}^2)$ by deriving an internal observability inequality and a Global Carleman estimate. Following the ideas contained in [25], the problem is reduced to prove an approximate theorem.

1. Introduction

The Korteweg–de Vries Burgers equation (KdV–B) was derived by Su and Gardner [29] for a wide class of nonlinear system in the weak nonlinearity and long wavelength approximation. This equation has been obtained when including electron inertia effects in the description of weak nonlinear plasma waves [13]. The KdV–Burgers equation has also been used in a study of wave propagation through liquid field elastic tube [16] and for a description of shallow water waves on viscous fluid. This model can be thought of as a composition of the KdV and Burgers equation,

$$u_t - \delta u_{xx} + u_{xxx} + u^p u_x = 0. \tag{1}$$

The equation (1) is one of the simplest evolution equations that features nonlinearity, dissipation, and dispersion. The special case $\delta = 0$ and $p = 1$ is the classical Korteweg–de Vries (KdV) equation which arises in modeling many practical situations involving wave propagation in nonlinear dispersive media. In spite of

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having many works dealing with the KdV equation in the existing literature, the same cannot be asserted to the KdV–Burgers equation. This lack of results becomes more evident when we are interested in the controllability or in the asymptotic behavior of its solutions. However, over the last years, a considerable number of stability issues concerning the KdV–B equation have received considerable attention. In what concerns to the boundary control and stabilization problems, we refer [1,3,9,10,14,15,17,20,26,28] and references therein. Nonetheless, to the best of our knowledge and already noticed in [7], there are few results about controllability for the KdV–B equation. Recently, M. Chen in [6] gets the existence of time optimal control and the null controllability of the Korteweg–de Vries–Burgers equation posed in a bounded domain under effect of a control acting locally in a subset of the domain.

Theorem A. *([6, Theorem 1.1]) Let $I = (0, L)$ with $L > 0$ and ω be nonempty open subset of I. Consider the KdV–B equation with external force v:*

$$\begin{cases} y_t - y_{xx} + y_{xxx} + yy_x = v\chi_\omega & \text{in } I \times (0, +\infty) \\ y(0,t) = y(L,t) = y_x(0,t) = 0 & \text{in } (0, +\infty), \\ y(x,0) = y_0(x) & \text{in } I. \end{cases} \tag{2}$$

Then, for any $y_0 \in L^2(I) \setminus \{0\}$ and any $M > 0$, there exist a time $T^* > 0$ and a control $v^* \in L^2(I \times (0, T^*))$ such that the solution y of (2) satisfies $y(\cdot, T^*) = 0$ and $\|v^*\|_{L^2(I\times(0,T^*))} = M$.

We are interested in the exact controllability results concerning a linearized Korteweg–de Vries Burgers equation posed in a *unbounded domain*. In this direction, also there is not too many results in the literature, see for instance [4], [12], [18], [19], [21] and [25]. In particular, Rosier in [25] studies the exact boundary controllability of the linearized KdV equation on the unbounded domain $\Omega = (0, \infty)$, where the control problem is discussed implicitly by considering the solution set without specifying the boundary conditions, such that the exact controllability does not hold for bounded energy solutions, i.e. for solutions in $L^\infty(0, T; L^2(0, \infty))$. His main result reads as follows:

Theorem B. *(Rosier [25, Thm. 1.3]) Let T, ε, b be positive numbers, with $\varepsilon < T$. Let $L^2(\Omega, e^{-2bx}dx)$ denote the space of (class of) measurable functions $u : \Omega \to \mathbb{R}$ such that $\int_0^\infty u^2(x)e^{-2bx}dx < \infty$. Let $u_0 \in L^2(\Omega)$ and $u_T \in L^2(\Omega, e^{-2bx}dx)$. Then, there exists a function*

$$u \in L^2_{loc}([0,T] \times [0,\infty]) \cap C([0,\varepsilon], L^2(\Omega)) \cap C([T-\varepsilon, T], L^2(\Omega, e^{-2bx}dx))$$

fulfilling

$$\begin{cases} u_t + u_x + u_{xxx} &= 0 \quad \text{in } \mathcal{D}'(\Omega \times (0,T)) \\ u|_{t=0} &= u_0, \\ u|_{t=T} &= u_T. \end{cases}$$

In Theorem B, u is locally square integrable. Actually, for a certain function u_0 in $L^2(0, \infty)$ and $u_T = 0$ a trajectory u as above cannot be found in $L^\infty(0, T, L^2(0, \infty))$ (see [25, Theorem 1.2]). It means that the bad behavior of the trajectories as $x \to \infty$ is the price to be paid for getting the exact controllability in the half space Ω. In the whole space, the same sort of results occurs for the heat and Schrodinger equations.

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