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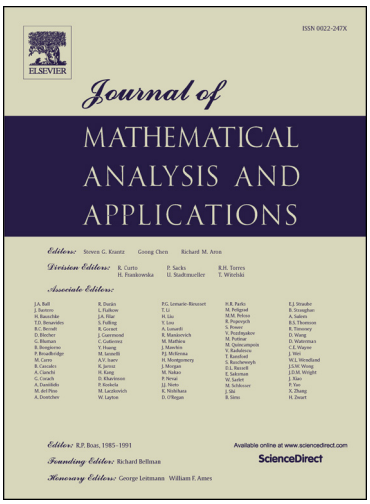

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Positive Solutions for the Kirchhoff-Type Problem Involving General Critical Growth – Part II: 3D Numerical Solutions

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Abstract

In this paper, we continue the study of the following Kirchhoff-type problem

$$\begin{cases} \left(a+\lambda\int_{\mathbb{R}^3}|\nabla u|^2dx+\lambda b\int_{\mathbb{R}^3}|u|^2dx\right)(-\Delta u+bu)=f(u), \text{ in } \mathbb{R}^3, \\ u\in H^1(\mathbb{R}^3),\ u>0, \text{ in } \mathbb{R}^3, \end{cases}$$

where $\lambda \geq 0$ is a parameter, a, b are positive constants and f reaches the critical growth. We use a scaling iterative algorithm to find numerical solutions to the problem on a large enough bounded domain with several particular nonlinearities f, including those with critical growth. We also study the behavior of the solutions as λ decreases to 0 in consonance with the theoretical study in Part I.
Keywords: Kirchhoff-type problem, positive solutions, critical nonlinearity, numerical solutions

1 Introduction

Part I of the paper deals with the existence of positive solutions for the following nonlinear Kirchhoff-type problem

$$\begin{cases} \left(a+\lambda\int_{\mathbb{R}^3}|\nabla u|^2dx+\lambda b\int_{\mathbb{R}^3}|u|^2dx\right)(-\Delta u+bu)=f(u), \text{ in } \mathbb{R}^3, \\ u\in H^1(\mathbb{R}^3),\ u>0, \text{ in } \mathbb{R}^3, \end{cases} \tag{1.1}$$

where $\lambda \geq 0$, a, b are positive constants and the general nonlinearity f reaches critical growth. The main results in Part I is outlined in the following. Assume the conditions

(H_1) $f \in C(\mathbb{R}_+, \mathbb{R}_+)$, $\mathbb{R}_+ = [0,\infty)$ and $\lim\limits_{s\to 0^+}\dfrac{f(s)}{s}=0$;