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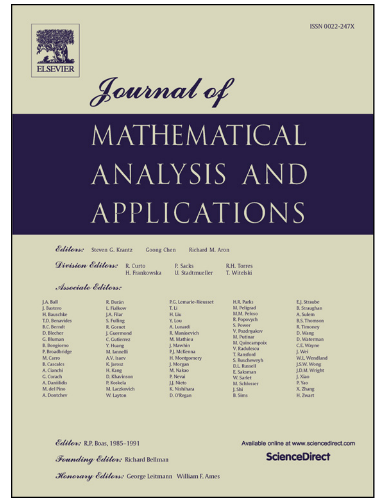

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GEOMETRICALLY DISTRIBUTED STIRLING WORDS AND STIRLING COMPOSITIONS

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ABSTRACT. A nonempty word w of finite length over the alphabet of positive integers is a *Stirling word* if for each letter i in w all entries between two consecutive occurrences of i (if these exist) are larger or equal to i. We derive an exact and also an asymptotic formula for the probability that a random geometrically distributed word of length n is a Stirling word. We also determine an asymptotic estimate for the number of compositions (called Stirling compositions) that satisfy this property. Moreover, we find generating functions and asymptotics formulas for statistics in Stirling compositions and geometrically distributed Stirling words, such as the number of distinct values and the size of the maximum part. The proofs make use of various techniques of advanced asymptotic analysis, including Mellin transforms and the saddle point method.

Keywords: Geometrically distributed Stirling words; Stirling compositions; Asymptotic formulas; Mellin transform

1. Introduction

In the last decades, several researchers have become interested in Stirling permutations and Stirling words over a finite alphabet (see [1, 4, 6]). In this paper, we derive the probability that a geometrically distributed word of length n is a Stirling word (for the case of set partitions, see [7]).

If $0 \le p \le 1$, then a discrete random variable X is said to be *geometric* if $P(X = i) = pq^{i-1}$ for all integers $i \ge 1$, where $q = 1 - p$. We will say that a word $w = w_1 w_2 \cdots$ over the alphabet of positive integers is *geometrically distributed* if the positions of w are independent and identically distributed geometric random variables. The research in geometrically distributed words has been a recent topic of study in enumerative combinatorics; see, e.g., [2, 3] and the references therein. A nonempty word w of finite length over the alphabet of positive integers is a *Stirling word* if for each letter i in w all entries between two consecutive occurrences of i (if these exist) are larger or equal to i.

In this paper, we study the generating functions for the probabilities of geometrically distributed Stirling words according to different statistics. In particular, we show that the probability P_n that a geometrically distributed word of length n is a Stirling word is given by

$$P_n = \frac{p^n}{1-q^n} \prod_{j=1}^{n-1} \left(1 + \frac{(n+1-j)q^j}{1-q^j}\right), \tag{1}$$

see Theorem 2.5.

We also study the closely related concept of *Stirling compositions* (see [5]). For a positive integer n, a *composition of* n is a word over the alphabet $\mathbb{N}$ of positive integers whose summands (letters) add up to n. Stirling compositions are those compositions that form a Stirling word. Note that Stirling compositions are precisely the 212-avoiding compositions (a 212-avoiding composition $\sigma_1\sigma_2\cdots\sigma_m$ is a composition such that there are no indices $1 \le i < j < i' \le m$ with $\sigma_i = \sigma_{i'} > \sigma_j$).

Moreover, we find generating functions for statistics in Stirling compositions and geometrically distributed Stirling words of length n, such as the number of distinct values and the size of the maximum part. In Section 3, by various techniques of advanced asymptotic analysis, including

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