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The exponential behavior of a stochastic globally modified Cahn–Hilliard–Navier–Stokes model with multiplicative noise

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ABSTRACT

In this article, we study the stability of weak solutions to a stochastic version of a globally modified coupled Cahn–Hilliard–Navier–Stokes model with multiplicative noise. The model consists of the globally modified Navier–Stokes equations for the velocity, coupled with an Cahn–Hilliard model for the order (phase) parameter. We prove that under some conditions on the forcing terms, the weak solutions converge exponentially in the mean square and almost surely exponentially to the stationary solutions. We also prove a result related to the stabilization of these equations.

1. Introduction

It is well accepted that the incompressible Navier–Stokes (NS) equation governs the motions of single-phase fluids such as air or water. On the other hand, we are faced with the difficult problem of understanding the motion of binary fluid mixtures, that is fluids composed by either two phases of the same chemical species or phases of different composition. Diffuse interface models are well-known tools to describe the dynamics of complex (e.g., binary) fluids, [16]. For instance, this approach is used in [2] to describe cavitation phenomena in a flowing liquid. The model consists of the NS equation coupled with the phase-field system, [3,15–17]. In the isothermal compressible case, the existence of a global weak solution is proved in [14]. In the incompressible isothermal case, neglecting chemical reactions and other forces, the model reduces to an evolution system which governs the fluid velocity v and the order parameter ϕ. This system can be written as a NS equation coupled with a convective Allen–Cahn equation, [16]. The associated initial and boundary value

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problem was studied in [16] in which the authors proved that the system generated a strongly continuous semigroup on a suitable phase space which possesses a global attractor. They also established the existence of an exponential attractor. This entails that the global attractor has a finite fractal dimension, which is estimated in [16] in terms of some model parameters. The dynamic of simple single-phase fluids has been widely investigated although some important issues remain unresolved, [26]. In the case of binary fluids, the analysis is even more complicate and the mathematical studied is still at it infancy as noted in [16]. As noted in [15], the mathematical analysis of binary fluid flows is far from being well understood. For instance, the spinodal decomposition under shear consists of a two-stage evolution of a homogeneous initial mixture: a phase separation stage in which some macroscopic patterns appear, then a shear stage in which these patters organize themselves into parallel layers (see, e.g. [21] for experimental snapshots). This model has to take into account the chemical interactions between the two phases at the interface, achieved using a Cahn–Hilliard approach, as well as the hydrodynamic properties of the mixture (e.g., in the shear case), for which a Navier–Stokes equations with surface tension terms acting at the interface are needed. When the two fluids have the same constant density, the temperature differences are negligible and the diffuse interface between the two phases has a small but non-zero thickness, a well-known model is the so-called "Model H" (cf. [18]). This is a system of equations where an incompressible Navier–Stokes equation for the (mean) velocity v is coupled with a convective Cahn–Hilliard equation for the order parameter ϕ, which represents the relative concentration of one of the fluids.

The long-time behavior of flows is a very interesting and important problem in the theory of fluid dynamic. As the vast literature shows [1,4,5,12,13,19,22,23,26,28], the problem has been receiving very much attention over the last three decades.

Another interesting question is to analyze the effects produced on a deterministic system by some stochastic or random disturbances appearing in the problem. This problem has been studied for the NS model, [6,7]. In [6], the authors studied the stability of the stationary solutions of the stochastic 2D NS equations. In particular, they proved that the weak solutions converge exponentially in the mean square and almost surely exponentially to the stationary solutions under some restrictions on the viscosity and the forcing terms. In [7], the authors generalized to the results of [6] to a class of dissipative nonlinear systems that include the 3D Lagrangian average NS equations.

Our work is motivated by the above references. We study the stability of weak solutions to the stochastic 3D globally modified CH-NS (GMCHNS) model with multiplicative noise. In particular, we proved that the weak solutions converge exponentially in the mean square and almost surely exponentially to the stationary solutions under some restrictions on the viscosity and the forcing terms. Let us note that the coupling between the Navier–Stokes and the Cahn–Hilliard systems makes the analysis of the control problem more involved.

The article is divided as follows. In the next section, we introduce the stochastic 3D GMCHNS model and its mathematical setting. The third section studies the stability of weak solutions. As in [6], applying the Itô formula, we study the stability of stationary solutions to the stochastic 3D GMCHNS model. We also prove in the fourth section a result related to the stabilization of these equations.

2. The stochastic GMCHNS model and its mathematical setting

2.1. Governing equations

In this article, we consider a modified version of the coupled CH-NS model with multiplicative noise. More precisely, we assume that the domain $\mathcal{M}$ of the fluid is a bounded domain in $\Re^3$. Then, we consider the following coupled CH-NS system

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