



Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa



# Competitive exclusion in a multi-strain virus model with spatial diffusion and age of infection <sup>☆</sup>

Xi-Chao Duan <sup>a,b,\*</sup>, Jun-Feng Yin <sup>a</sup>, Xue-Zhi Li <sup>c</sup>, Maia Martcheva <sup>d</sup>

<sup>a</sup> School of Mathematical Sciences, Tongji University, Shanghai 200092, China

<sup>b</sup> College of Information Technology, Shanghai Ocean University, Shanghai 201306, China

<sup>c</sup> Department of Mathematics and Physics, Anyang Institute of Technology, Anyang 455000, China

<sup>d</sup> Department of Mathematics, University of Florida, 358 Little Hall, PO Box 118105, Gainesville, FL 32611-8105, United States

## ARTICLE INFO

### Article history:

Received 8 September 2016

Available online xxxx

Submitted by Y. Du

### Keywords:

Age of infection

Multi-strain

Spatial diffusion

General incidence function

Reproduction number

Competitive exclusion

## ABSTRACT

In this paper, a multi-strain virus dynamic model with spatial diffusion, age of infection and general incidence function is formulated. The well-posedness of the initial-boundary value problem of the model in the bounded domain  $\Omega \subset \mathbb{R}^n$  is analyzed. By constructing a suitable Lyapunov functional, the global stability of the uninfected steady state is established if all reproduction numbers are smaller or equal to one. It is shown that if  $\mathcal{R}_i$ , the reproduction number corresponding to strain  $i$  is larger than one, the steady state corresponding to strain  $i$  exists, if  $\mathcal{R}_1 > 1$  is the maximal reproduction number, the steady state  $E_1$  corresponding strain one is globally stable. That is, competitive exclusion occurs and strain one eliminates all other strains.

© 2017 Elsevier Inc. All rights reserved.

## 1. Introduction

Since the time of Gause [8], scientists have been fascinated by the principles that govern species coexistence and competitive exclusion. Coexistence and competitive exclusion has been studied in the context of ecological models, epidemiological models, within-host models and other settings. The first rigorous proof of the competitive exclusion principle was given by Bremermann and Thieme [1] in epidemiological setting. Bremermann and Thieme showed that in the most basic multi-strain SIR model the strain with the largest reproduction number outcompetes the remaining strains and persists in the population. This principle was

<sup>☆</sup> Supported partially by the Doctoral Scientific Research Foundation of Shanghai Ocean University A2-0203-17-100323; X. Li is supported partially by the Natural Science Foundation of China (11771017) and Plan for Scientific Innovation Talent of Henan Province (144200510021); M. Martcheva is supported partially through grants National Science Foundation DMS-1220342 and DMS-1515661.

\* Corresponding author at: School of Mathematical Sciences, Tongji University, Shanghai 200092, China.

E-mail addresses: [xcduan82@126.com](mailto:xcduan82@126.com) (X.-C. Duan), [yinjf@tongji.edu.cn](mailto:yinjf@tongji.edu.cn) (J.-F. Yin), [xzli66@126.com](mailto:xzli66@126.com) (X.-Z. Li), [maia@ufl.edu](mailto:maia@ufl.edu) (M. Martcheva).

extended to age-since-infection structured epidemic models with environmental transmission by Martcheva and Li [13]. Regarding epidemic models with diffusion, Martcheva and Tuncer [20] showed that in a two-strain SIS model with diffusion and spatially non-homogeneous coefficients coexistence may occur. Wu et al. [22] investigated the topic further and showed competitive exclusion in some special cases with constants coefficients.

Within-host models have also received significant attention. De Leenheer and Smith [5] first established the global stability of the infection equilibrium in the most widely used HIV/HCV mathematical model using the so-called “sector condition”. Global competitive exclusion was then proved in a multi-strain ODE version by De Leenheer and Pilyugin [4]. This result was also extended to a multi-strain age-since-infection structured model by Browne [2]. Variations of the within-host HIV model with diffusion have also been considered. Stancevic et al. [17] consider a version with simple recruitment but reaction and diffusion terms in all classes. They find Turing instability in the model. More recently, a spatial version of the classical within-host HIV model with diffusion and spatially heterogeneous coefficients was more thoroughly analyzed and global asymptotic behavior of the solutions was established [16]. To our knowledge, spatial multi-strain versions of the model have not been considered.

Versions of the basic within-host HIV ODE model with general incidence have also been considered. Tian and Liu [19] consider the model with simple recruitment and general incidence. They find that the infection equilibrium is globally stable whenever it exists. The global stability of a diffusion version of the within-host model with general incidence was recently studied by McCluskey and Yang [15].

In this paper we consider a multi-strain version of the basic within-host HIV model with general incidence, age-since-infection structure of infected T cells, and diffusion of the virus. We aim to show global competitive exclusion under these conditions, thus extending prior results. In the next section, we introduce the multi-strain model with age-since-infection structure, diffusion and general incidence. In section 3 we derive some preliminary results related to the properties of the flow. In section 4 we show the global stability of the infection-free state. In section 5 we consider the uniform persistence of the system and we prove the competitive exclusion principle. Section 6 discusses some applications and section 7 summarizes our results.

**2. The multi-strain model**

In this section we introduce our multi-strain model with age-since-infection structure, diffusion and general incidence. The model is given below.

$$\begin{cases} \frac{\partial u}{\partial t} = r - mu(x, t) - \sum_{i=1}^n f_i(u(x, t), V_i(x, t)), \\ \frac{\partial w_i(x, \theta, t)}{\partial \theta} + \frac{\partial w_i(x, \theta, t)}{\partial t} = -\delta_i(\theta)w_i(x, \theta, t), \quad i = 1, \dots, n, \\ \frac{\partial V_i}{\partial t} = D_i \Delta V_i + \int_0^\infty p_i(\theta)w_i(x, \theta, t)d\theta - c_i V_i(x, t), \end{cases} \tag{2.1}$$

for  $t > 0, x \in \Omega$ , with homogeneous Neumann boundary conditions

$$\frac{\partial V_i}{\partial \nu} = 0, \quad t > 0, \quad x \in \partial\Omega, \quad i = 1, \dots, n, \tag{2.2}$$

and the initial and boundary conditions:

$$\begin{cases} w_i(x, 0, t) = f_i(u(x, t), V_i(x, t)), \quad x \in \bar{\Omega}, \quad i = 1, \dots, n, \\ u_i(x, 0) = u_{i0}(x), \quad w_i(x, \theta, 0) = w_{i0}(x, \theta), \quad V_i(x, 0) = V_{i0}(x), \quad x \in \bar{\Omega}. \end{cases} \tag{2.3}$$

Download English Version:

<https://daneshyari.com/en/article/8900128>

Download Persian Version:

<https://daneshyari.com/article/8900128>

[Daneshyari.com](https://daneshyari.com)