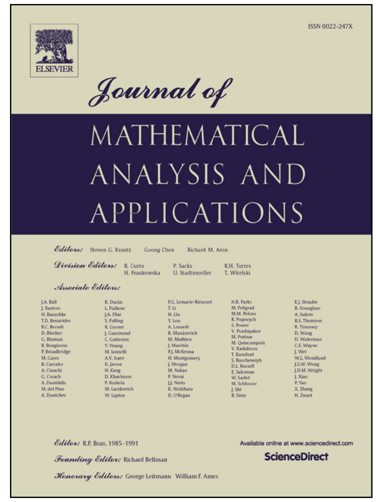

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Strong mixing Gaussian measures for chaotic semigroups

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Abstract

In this paper we will be concerned with the problem of the existence of an invariant mixing measure considering its connection with the chaotic behavior of linear semigroups on separable Banach spaces. We first prove an identity characterizing invariant Gaussian measure involving its covariance operator and the infinitesimal generator of the semigroup . This gives an answer to a question raised by Rudnicki in his inspiring review paper [35]. Under suitable conditions, we use the proved identity to give an invariant mixing Gaussian measure as distribution of a Wiener integral.

1 Introduction

It is well known that the essence of deterministic chaos is the sensitive dependence on initial conditions. The last property means that small variations of the initial state may produce large variations in the long term behavior of the studied dynamical system. As a consequence, the evolution becomes unpredictable despite the deterministic description of the dynamic. It is commonly believed that chaos can occur only within nonlinear phenomena but models arising from biology, physics and other fields show that linear infinite dimensional systems can exhibit chaotic behavior exactly like nonlinear ones, see for instance [1,2,10,20,22,23,27,34] and references therein. For a systematic study of linear chaos see the seminal paper [17] and the survey book [24]. In almost all cases, the definition of a chaotic system introduced by Devaney [19], is proved to be the most convenient. One of the features of the chaotic dynamic, following this definition, is the so called topological transitivity. This property is a sign of complexity preventing to reduce the study to simple subsystems. Let us formulate this concept for a linear C_0-semigroup $T(\cdot)$ on a Banach space E. We say that $T(\cdot)$ is *topologically transitive* if for any two nonempty open subsets U, V of E there exists $t > 0$ such that $T_t(U) \cap V \neq \emptyset$. Furthermore, if E is assumed to be separable then this is

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