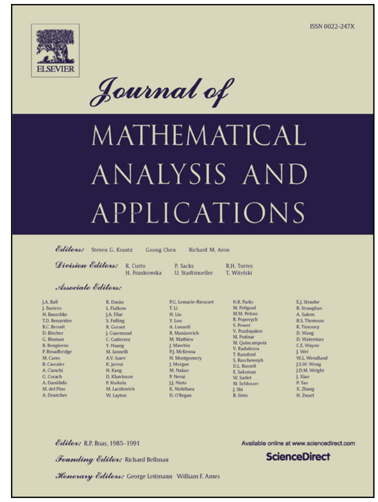
Accepted Manuscript



Reflexivity is equivalent to stability of the almost fixed point property

H. Fetter, M. Japón, J. Villada Bedoya

PII: S0022-247X(17)31000-4
DOI: https://doi.org/10.1016/j.jmaa.2017.11.007
Reference: YJMAA 21805

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 23 June 2017

Please cite this article in press as: H. Fetter et al., Reflexivity is equivalent to stability of the almost fixed point property, *J. Math. Anal. Appl.* (2018), https://doi.org/10.1016/j.jmaa.2017.11.007

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

ACCEPTED MANUSCRIPT

Reflexivity is equivalent to stability of the Almost Fixed Point Property

H. Fetter[a], M. Japón[b], J. Villada Bedoya[a]

[a]Centro de investigación en Matemáticas (CIMAT), Jalisco s/n, Valenciana, CP 36240 Guanajuato, Gto., Mexico.
[b]Facultad de Matemáticas, Universidad de Sevilla. c/Tarfia s/n. 41012-Sevilla. Spain.

Abstract

In this work we show that the family of closed convex sets with the almost fixed point property is not stable under renormings for non-reflexive Banach spaces. This, together with a result by Reich, shows that a Banach space is reflexive if and only if it has the same family of closed convex sets with the almost fixed point property for every equivalent norm.

Keywords: Reflexive Banach spaces, equivalent norms, nonexpansive mappings, almost fixed point property AFPP, stability, directionally bounded sets
MSC: 47H09, 47H10, 47H30

1. Introduction and Preliminaries

A closed convex subset C of a Banach space $(X, \|\cdot\|)$ is said to have the almost fixed point property (AFPP) if $\inf\{\|x - Tx\| : x \in C\} = 0$ for every nonexpansive mapping $T : C \to C$, where nonexpansiveness means $\|Tx - Ty\| \leq \|x - y\|$ for every $x, y \in C$. It is well-known that every closed convex subset has the AFPP whenever it is bounded. Otherwise, closed convex unbounded sets with the AFPP can be characterized by using geometrical properties. A set C is linearly bounded if its intersection with every line is bounded. For reflexive Banach spaces S. Reich [6] proved the following:

Theorem 1.1. *Let X be a reflexive Banach space and let C be a closed convex subset of X. Then C has the AFPP if and only if C is linearly bounded.*

However, every non-reflexive Banach space contains a closed convex linearly bounded set C that fails to have the AFPP [7, Proposition 3.5 and Theorem 3.2], which shows that linear boundedness does not characterize the AFPP unless the Banach space is reflexive.

For general Banach spaces, I. Shafrir [7] introduced the concept of directionally bounded set: A convex unbounded subset $C \subset X$ is said to be directionally bounded if for every $(x_n) \subset C$ with $\lim_n \|x_n\| = +\infty$ and for every $f \in X^*$ with $\|f\| = 1$,

$$\limsup_n f\left(\frac{x_n}{\|x_n\|}\right) < 1.$$

Theorem 1.2. *[7] A closed convex set of a Banach space satisfies the AFPP if and only if it is directionally bounded.*

In particular, every closed convex directionally bounded subset is linearly bounded, but these two concepts are not equivalent beyond reflexivity assumptions.

In this paper we study the stability under renormings of the class of closed convex sets with the AFPP. Since the concept of being linearly bounded is preserved under renormings, the class of closed convex subsets with the AFPP is invariant for every equivalent norm in reflexive Banach spaces. However, in the absence of reflexivity, we can pose the

Email addresses: fetter@cimat.mx (the corresponding author) (H. Fetter), japon@us.es (M. Japón), jeimer.villada@cimat.mx (J. Villada Bedoya)

Download English Version:

https://daneshyari.com/en/article/8900136

Download Persian Version:

https://daneshyari.com/article/8900136

Daneshyari.com