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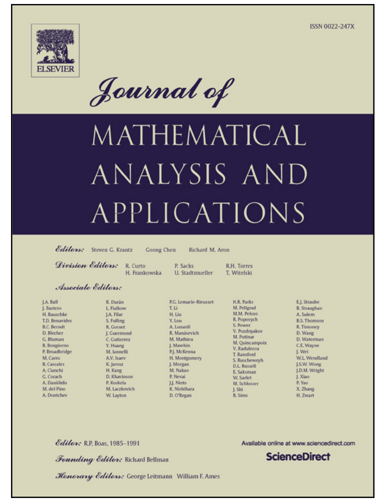

Weak solutions to non-homogeneous boundary value problems for time-fractional diffusion equations

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Weak solutions to non-homogeneous boundary value problems for time-fractional diffusion equations

Masahiro Yamamoto *

Abstract

We discuss an initial-boundary problem for a time-fractional diffusion equation with non-zero Dirichlet boundary values which belong to L^2 in time t and to a Sobolev space of negative order in space and prove the unique existence of weak solutions and a priori estimates. The proof is based on the Caputo fractional derivative in Sobolev spaces and the transposition method. We show one application to the existence of solution to an optimal control problem.

§1. Introduction

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with boundary $\partial\Omega$ of C^2-class and let $\nu = \nu(x) = (\nu_1, ..., \nu_n)$ be the unit outward normal vector to $\partial\Omega$ at x. In this paper we consider an initial-boundary problem for a time-fractional diffusion:

$$\partial_t^\alpha u(x,t) = -Lu(x,t), \quad x \in \Omega,\ 0 < t \le T, \tag{1.1}$$

$$u(x,t) = g(x,t), \quad x \in \partial\Omega,\ 0 < t < T, \tag{1.2}$$

$$u(x,0) = 0, \quad x \in \Omega. \tag{1.3}$$

Throughout this paper, we assume $0 < \alpha < 1$, and by $\partial_t^\alpha u$, we denote the Caputo derivative, which can be defined for $w \in C^1[0,T]$ by

$$\partial_t^\alpha w(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} \frac{dw}{ds}(s)ds, \quad 0 < t \le T, \quad 0 < \alpha < 1, \tag{1.4}$$

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