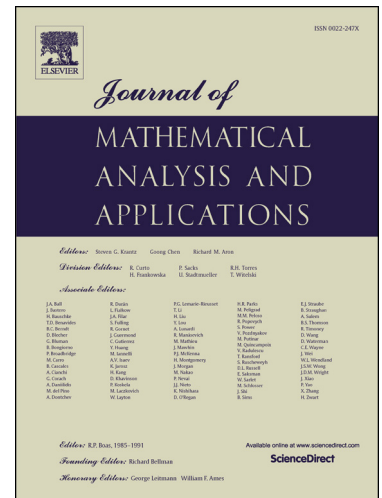




Strongly essential set of Ky Fan's points and the stability of Nash equilibrium ☆

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Abstract

In this paper, a stronger perturbation of inequality functions and set-valued mappings is proposed by means of the Hausdorff upper semi-metric, which includes the perturbations defined by sup-norm of function and the maximum Hausdorff metric of section mapping. Based on this perturbation, a class of strongly essential sets of Ky Fan's points is introduced, and the existence of the strongly essential component of Ky Fan's points is proved. As an application, we use the equivalence of Nash equilibrium with Ky Fan's points to obtain the existence of the strongly essential component of equilibrium, which not only give a class of stronger essential sets, but also provide a method to discuss the stability with respect to the perturbation of strategic sets.

Keywords: Hausdorff metric, Ky Fan's point, Nash equilibrium, Strong essential set, Strong essential component

1. Introduction

Let X be a nonempty compact convex set of linear topological space and $f : X \times X \rightarrow R$ be a real valued function. We introduce the following Ky Fan's inequality problem (see, [1]):

(KF) find $x^* \in X$ such that $f(x^*, y) \leq 0$ for each $y \in X$.

Ky Fan's inequality problem is a equivalent form of Ky Fan's section theorem (see, [2]). The above function $f : X \times X \rightarrow R$ is called a inequality function, and $x^* \in X$ is called a Ky Fan's point.

Ky Fan's inequality theorem plays a very important role in the research of nonlinear and convex analysis. Because of the application of Ky Fan's inequality in optimization, convex analysis, variational inequality, optima control, fixed point problem and economics, Ky Fan's inequality was extended to various forms, such as implicit variational inequality, equilibrium problem, vector variational inequality and mixed implicit variational inequality (see, e.g., Refs. [3, 4, 5, 6, 7, 8, 9, 10, 11, 11, 12]).

Since Ky Fan's point has some equivalence with Nash equilibrium, it becomes a powerful tool to study the noncooperative game problem. Nash equilibrium is a core concept of noncooperative games, and it has broad applicability, which extends well beyond economics and other behavioral sciences. However, it becomes clear that the concept of equilibrium has some serious drawbacks that limit its usefulness. To start with, there is often more than one equilibrium, and, in some cases, there is a very large (or even infinite) number of them. Multiple equilibria makes players not be clear about which one to focus on, since there is no reasonable way of deciding which equilibrium will end up being selected. In addressing multiplicity of Nash equilibria, game theorists have examined a variety of arguments that refine the set of equilibrium (see, e.g., Refs. [13, 14, 15, 16, 17, 18]). The refinement of equilibrium should follow certain rational principle, so a reasonable approach for refinement, as used in many concepts is to select equilibrium that is "stable" to slight perturbation caused from the uncertainty in the game. Therefore, the stability has become an

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