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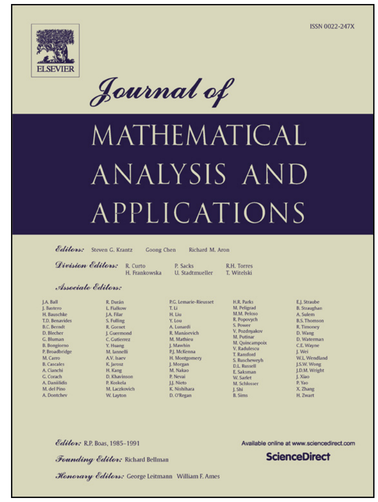

Symplectic rigidity of real and complex polydiscs

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SYMPLECTIC RIGIDITY OF REAL AND COMPLEX POLYDISCS

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Abstract. In $\mathbb{R}^{2n}$ with its standard symplectic structure, the complex polydisc, $\mathbb{D}^{2n}_{\mathbb{C}}(r)$, is constructed as the product of n open complex discs of radius r. When $n = 2$, the real polydisc, $\mathbb{D}^4_{\mathbb{R}}(r)$, is constructed as the product of 2 open real/Lagrangian discs of radius r. Sukhov and Tumanov recently showed that $\mathbb{D}^4_{\mathbb{C}}(1)$ and $\mathbb{D}^4_{\mathbb{R}}(1)$ are not symplectically equivalent. We extend this result in two ways. First we give the necessary and sufficient conditions for an orthogonal image of $\mathbb{D}^4_{\mathbb{C}}(1)$ to be symplectically equivalent to $\mathbb{D}^4_{\mathbb{C}}(1)$. Second, we show that for all $r \geq 1$ and $n \geq 1$, $\mathbb{D}^4_{\mathbb{R}}(1) \times \mathbb{D}^{2n-4}_{\mathbb{C}}(r)$ is not symplectically equivalent to $\mathbb{D}^4_{\mathbb{C}}(1) \times \mathbb{D}^{2n-4}_{\mathbb{C}}(r)$.

1. Introduction

The problem of symplectic rigidity has been studied for a long time. The first striking result was obtained by Gromov [3], which states that one can symplectically embed a sphere into a cylinder only if the radius of the sphere is less than or equal to the radius of the cylinder. Following Gromov's work, many results on symplectic rigidity were obtained for various domains. For example, McDuff [5] studied when a 4-dimensional ellipsoid can be symplectically embedded in a ball; Guth [4] gave an asymptotic result on when a polydisc $\mathbb{D}^2_{\mathbb{C}}(r_1) \times \cdots \times \mathbb{D}^2_{\mathbb{C}}(r_n)$ can be symplectically embedded into another.

Sukhov and Tumanov [7] applied techniques in classical complex analysis to a problem of symplectic rigidity. They showed that the real bi-disc $\mathbb{D}^4_{\mathbb{R}}(1) = \{(z_1, z_2) \in \mathbb{C}^2 : |x_1|^2 + |x_2|^2 < 1, |y_1|^2 + |y_2|^2 < 1\}$ cannot be symplectically embedded into the complex cylinder $\mathbb{D}^2_{\mathbb{C}}(1) \times \mathbb{C}$ of radius 1. If we consider the real bidisc as obtained from a non-holomorphic change of coordinates

$$T_0 : (x_1, y_1, x_2, y_2) \mapsto (x_1, x_2, y_1, y_2)$$

of $\mathbb{D}^4_{\mathbb{C}}(1)$, then the result of Sukhov and Tumanov shows that $T_0(\mathbb{D}^4_{\mathbb{C}}(1))$ is not symplectomorphic to $\mathbb{D}^4_{\mathbb{C}}(1)$ itself.

In this paper, we apply the complex analysis techniques used by Sukhov and Tumanov [7] to solve the problem of symplectic rigidity on different domains: real bidisc and its modifications.

Let $x_1, y_1, \ldots, x_n, y_n$ be the standard coordinates on the $2n$-dimensional Euclidean space $\mathbb{R}^{2n} \cong \mathbb{C}^n$, the standard symplectc form on the space is given by $dx_1 \wedge dy_1 + \cdots + dx_n \wedge dy_n$. All symplectic embeddings considered in this paper will be with respect to the standard symplectic form on $\mathbb{R}^{2n}$, unless otherwise specified. For $n > 1$, define the real $2n$-dimensional complex n-disc of radius r by

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