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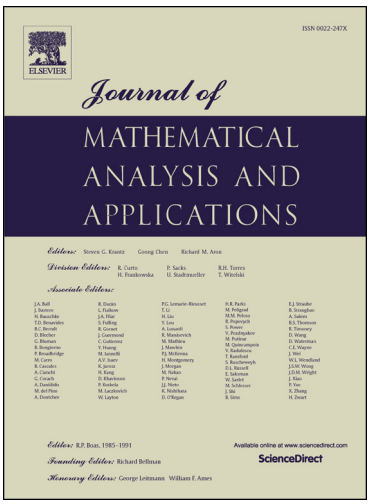

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A Riemann–Hilbert approach for a new type coupled nonlinear Schrödinger equations

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Abstract. We develop a Riemann–Hilbert approach to the Cauchy problem on the line for a new type of coupled nonlinear Schrödinger (CNLS) equations

$$\mathrm{i}q_{1,t} + q_{1,xx} + 2(|q_1|^2 - 2|q_2|^2)q_1 - 2q_2^2 q_1^* = 0,$$
$$\mathrm{i}q_{2,t} + q_{2,xx} + 2(2|q_1|^2 - |q_2|^2)q_2 + 2q_1^2 q_2^* = 0.$$

This approach allows us to give a representation of the solution to the Cauchy problem of the CNLS equations in terms of the solution of a 4×4 Riemann–Hilbert problem formulated in the complex k-plane. Due to the energy conservation law of above system is $\int_{-\infty}^{+\infty}(|q_1|^2 - |q_2|^2)dx$, it is difficult to obtain a solution for this system by using the energy estimate method of PDE's. Therefore, this approach efficiently provides a new way in studying the nonlinear problems that PDE's theory can't solve. Furthermore, this representation is then used for retrieving the soliton solutions.

Keywords: Coupled nonlinear Schrödinger equations, Riemann–Hilbert problem, Inverse scattering transform, Solitons.

1 Introduction

The celebrated nonlinear Schrödinger (NLS) equation

$$\mathrm{i}q_t + q_{xx} - 2\lambda|q|^2 q = 0, \quad \lambda = \pm1, \tag{1.1}$$

as one of the universal nonlinear integrable models that describe the evolution of slowly varying packets of quasi-monochromatic waves in weakly nonlinear dispersive media, has

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