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Instantaneous blow-up versus local solvability for one problem of propagation of nonlinear waves in semiconductors ☆

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ABSTRACT

This work develops the theory of the blow-up phenomena for one Sobolev problem that arises in the theory of propagation of nonlinear waves in semiconductors. This problem is considered as 1) the Cauchy problem, 2) the initial–boundary value problem on the half-line and 3) the initial–boundary value problem on a segment. It was shown that in the first two cases the problem does not have weak solution even locally in time, but in the third case the problem has the classical solution that exists at least locally in time. The upper estimate of solvability time for classical solution in the third case is obtained. This analytical *a priori* information was used in the numerical experiment, which is able to determine the process of the solution's blow-up more accurately.

1. Introduction

For the first time the "complete blow-up" phenomena were shown for the equation

$$-\Delta u = |x|^{-2}u^2, \quad u \geqslant 0, \quad x \in \Omega\backslash\{0\} \subset \mathbb{R}^N \tag{1.1}$$

in the work of H. Brezis and X. Cabre [7]. Then, for linear parabolic equation with singular potential the "instantaneous blow-up" phenomenon was obtained in [8]. For the singular nonlinear parabolic equation

$$u_t - \Delta u = |x|^{-2}u^2, \quad u \geqslant 0, \quad x \in \Omega\backslash\{0\} \subset \mathbb{R}^N, \quad t > 0 \tag{1.2}$$

the question about instantaneous blow-up was considered for the first time by F.B. Weissler [24].

It should be noted that in these works the comparison method was used and the proof technique was sufficiently difficult. In the work of E. Mitidieri and S.I. Pokhozhaev (see [20] and its bibliography) the

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results about complete and instantaneous blow-up for equations of higher order were obtained by simpler and effective method of non-linear capacitance.

In the following, instantaneous blow-up in nonlinear parabolic and hyperbolic equations was considered by V.A. Galaktionov and J.L. Vazquez [12], J.A. Goldstein and I. Kombe [14], Y. Giga and N. Umeda [13], E.I. Galakhov [10,11]. Note, that in some works the method based on the comparison principle was used (for parabolic equations). But in the works of E.I. Galakhov the method of S.I. Pokhozhaev, based on the method of non-linear capacitance, was developed. This method allows to get results about sufficient conditions of unsolvability for both parabolic and hyperbolic equations, including equations of higher order (not Sobolev equations).

For the first time the question about instantaneous blow-up in nonclassical Sobolev equations was obtained in [9]. In this work the following problem was considered:

$$\frac{\partial}{\partial t}(u_{xx}+u)=u_{xx}, \quad u(x,0)=u_0(x), \quad u(0,t)=u(l,t), \quad l>0. \tag{1.3}$$

As the corollary fact of Theorem 4.1 of the work [9], the result about nonexistence of a bounded solution of this problem was obtained for an indefinitely small time interval with condition that $l \in (0,\pi]$. This result is stipulated by the fact that under the time derivative the operator $\partial_x^2 + I$ is situated. Further, such results were appearing in the study of linear equation of Sobolev type

$$\frac{\partial}{\partial t}(\Delta u+\lambda u)+\Delta u=0 \quad \text{for} \quad \lambda>0, \quad x\in\Omega\subset\mathbb{R}^N,$$

in the case when the number λ falls on the spectrum of operator Δ in a bounded region Ω (see survey [23]). Particularly, at this survey the method of singular semigroup for study of linear Sobolev equations with singular operator with higher derivative was recited. In the sequel, the phenomena of the instantaneous blow-up for linear and nonlinear equations of Sobolev type have not been considering. The reason was that researchers were interested in the question about sufficient conditions of existence of the solution. In the work [1] the study of global unsolvability of Sobolev equations with fraction order of time derivative was performed. The new result that was obtained in the work [18] is, for example, that in the equation

$$\frac{\partial^2}{\partial t^2}\Delta u+\Delta u+|u|^q=0, \quad u(x,0)=u_0(x), \quad u'(x,0)=u_1(x),$$

there are no singular coefficients of the form $|x|^{-\alpha}$ or $t^{-\beta}$, the initial functions belong to the class $\mathbb{C}_0^\infty(\mathbb{R}^N)$, and there is no solution of the equation even locally in time under the condition

$$1<q\leqslant q_{kr}=\begin{cases} N/(N-2), & \text{if} \quad N\geqslant 3,\\ +\infty, & \text{if} \quad N=1,2.\end{cases} \tag{1.4}$$

In this paper we consider the Cauchy problem, the initial–boundary value problem on the half-line and the initial–boundary value problem on a segment for the equation

$$\frac{\partial^2 u}{\partial t\partial x}+\frac{\partial}{\partial x}\left(|u|^{p-2}u\right)=|u|^q,$$

which has certain physical meaning. We obtained the following results: the Cauchy problem and the problem on the half-line do not have weak solutions even locally in time, but the classical solution of this problem on the segment exists locally in time (also we obtained sufficient blow-up conditions of weak solution in finite time for the considered problem).

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