Inter-species competition and chemorepulsion

J. Ignacio Tello [a,b,*], Dariusz Wrzosek [c]

[a] *Depto. Matemática Aplicada a las T.I.C., ETSI Sistemas Informáticos, Universidad Politécnica de Madrid, 28031, Madrid, Spain*
[b] *Center for Computational Simulation, Universidad Politécnica de Madrid, 28660 Boadilla del Monte, Madrid, Spain*
[c] *Inst. Applied Mathematics and Mechanics, University of Warsaw, Banacha 2, 02-097 Warszawa, Poland*

A R T I C L E   I N F O

A B S T R A C T

We study an extension of the Lotka–Volterra competition model in which one of the competing species avoids encounters with rivals by means of chemorepulsion—a repulsive reaction to the scent of competitors. The evolution of the species densities is described in terms of parabolic equations with cross-diffusive and competitive terms. The chemical signal used to detect rivals may diffuse in the environment leading to a fully parabolic problem or to a parabolic–elliptic system if the diffusion of the chemical is dominant. The most difficult case from the viewpoint of well-posedness and global existence is the parabolic–ODE system which corresponds to the case when the chemical signal does not diffuse in the environment. We prove global existence of solutions in the three cases for any space dimension for a wide range of parameters and initial conditions. For the parabolic–elliptic case, the moving rectangles method is adapted to prove the converge to a constant steady state for some range of parameters and initial data. Linear stability analysis indicates that the constant steady state in the fully parabolic case loses stability when the strength of chemorepulsion is too high and that sufficiently low degradation rate of chemorepellent stabilizes the constant steady state.

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1. Introduction

We consider an extension of the classical model of interspecies competition due to Lotka [14] and Volterra [27] in which individuals belonging to both competing population are assumed to disperse randomly in the region which they jointly occupy. Moreover individuals of the first species try to avoid encounters with competitors by means of negative chemotaxis (chemorepulsion)—a chemo-sensory reaction to the scent of rivals. Depending on the qualitative properties of the chemical signal and the environment, the chemical (e.g. volatile odorants, allelochemicals or kairomones) may diffuse in the environment or it may be considered as

* Corresponding author at: Depto. Matemática Aplicada a las T.I.C., ETSI Sistemas Informáticos, Universidad Politécnica de Madrid, 28031, Madrid, Spain.
E-mail addresses: j.tello@upm.es (J.I. Tello), D.Wrzosek@mimuw.edu.pl (D. Wrzosek).

a scent mark deposited in the environment (e.g. urinary proteins) in the case when it is not diffusive. Unlike visual or acoustic signals, olfactory signal persists in the absence of the signaler often over extended period of time. Scent marks are widely used among terrestrial vertebrates in the context of territory marking and defense (see e.g. [7]). They can be used to provide information to conspecifics or to interspecies competitors (see [5] and [30]). Denoting the densities of competing species by u and v and the density of the chemical signal (e.g. volatile odorant) by w the model of interspecies competition with chemorepulsion reads

$$u_t = \text{div}(d_u \nabla u + \chi u \nabla w) + \mu_1 u(1 - u - a_1 v), \qquad x \in \Omega, \ t > 0 \tag{1.1}$$

$$v_t = d_v \Delta v + \mu_2 v(1 - v - a_2 u), \qquad x \in \Omega, \ t > 0 \tag{1.2}$$

$$\tau w_t = d_w \Delta w - \lambda w + \alpha v, \qquad x \in \Omega, \ t > 0 \tag{1.3}$$

where $d_u > 0$, $d_v > 0$, $d_w \geq 0$, are the diffusion coefficients, $\chi > 0$ is the chemorepulsion coefficient, $\mu_1 > 0$ and $\mu_2 > 0$ are the population growth rates and $a_i > 0$ (for $i = 1, 2$) are the coefficients which describe the strength of competition. The functions u and v are defined on $\Omega \times (0, +\infty)$ where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a bounded domain with smooth boundary. The system is supplemented by the boundary conditions

$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = d_w \frac{\partial w}{\partial \nu} = 0, \qquad x \in \partial \Omega \tag{1.4}$$

and initial data

$$u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x), \quad w(x, 0) = w_0(x) \quad x \in \Omega. \tag{1.5}$$

Similarly to the Patlak–Keller–Segel model of chemotaxis (see [20] and [9]), the flux of the first species J_u splits into two components, a pure diffusive flux J_{diff} and a chemotactic one J_{chem} to obtain

$$J_u = J_{diff} + J_{chem} = -d_u \nabla u - \chi u \nabla w.$$

The sign "$-$" in front of ∇w corresponds to a negative taxis, i.e. the movement of individuals towards decreasing concentration of the signaling chemical secreted by the individuals from the first species. The coefficients α and λ in (1.3) describe the rates of signal production and signal degradation respectively. By this indirect mechanism the first species try to avoid encounters with rivals of the second species. We consider three different cases for the model:

1. Fully parabolic system ($\tau = 1$ and $d_w > 0$) when the chemical signal disperses by diffusion in the environment and satisfies a second order equation of parabolic type;
2. Parabolic–elliptic system ($\tau = 0$ and $d_w > 0$) which results from assuming the quasi-stationary approximation when the dynamics of signaling chemical is much quicker then that of species densities and the equation for the chemical is reduced to an elliptic equation;
3. Parabolic–ODE system ($\tau = 1$, $d_w = 0$) refers to the case when the chemical signal is not diffusive. The chemical signal is assumed to be deposited at spots occupied by the second species and such an olfactory cue plays the role of chemorepellent for the first species (see e.g. [7] for terrestrial ecosystems).

The first or second scenario takes place for instance in an aquatic environment (see e.g. [21], [5] and [30]) while the third in terrestial one (see e.g. [7]).