J. Math. Anal. Appl. 459 (2018) 320–349

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Smoothing effect and Cauchy problem for radially symmetric homogeneous Boltzmann equation with Debye–Yukawa potential of Shubin class initial datum

Hao-Guang Li *, Yueyun Shang

School of Mathematics and Statistics, South-Central University for Nationalities, 430074, Wuhan, PR China

A R T I C L E   I N F O

Article history:
Received 8 March 2017
Available online 24 October 2017
Submitted by X. Zhang

Keywords:
Cauchy problem
Boltzmann equation
Debye–Yukawa potential
Shubin class initial datum

A B S T R A C T

In this paper, we study the Cauchy problem for the radially symmetric homogeneous non-cutoff Boltzmann equation with Debye–Yukawa potential, the initial datum belongs to Shubin type space of the negative index which can be characterized by spectral decomposition of the harmonic oscillator, and it is a small perturbation of Maxwellian distribution. The Shubin type space of negative index contains the probability measures. Based on the spectral decomposition, we construct the weak solution with Shubin type class initial datum and prove the smoothing effect for the solution to this Cauchy problem.

Contents

* Corresponding author.
E-mail addresses: lihaoguang@mail.scuec.edu.cn (H.-G. Li), shangyueyun1980@163.com (Y. Shang).

https://doi.org/10.1016/j.jmaa.2017.10.053
0022-247X/

1. Introduction

In this work, we consider the spatially homogeneous Boltzmann equation

$$\begin{cases} \partial_t f = Q(f, f), \\ f|_{t=0} = f_0 \geq 0, \end{cases} \tag{1.1}$$

where $f = f(t, v)$ is the density distribution function depending on the variables $v \in \mathbb{R}^3$ and the time $t \geq 0$. The Boltzmann bilinear collision operator is given by

$$Q(g, f)(v) = \int_{\mathbb{R}^3} \int_{\mathbb{S}^2} B(v - v_*, \sigma)(g(v'_*)f(v') - g(v_*)f(v))dv_* d\sigma,$$

where for $\sigma \in \mathbb{S}^2$, the symbols v'_* and v' are abbreviations for the expressions,

$$v' = \frac{v + v_*}{2} + \frac{|v - v_*|}{2}\sigma, \quad v'_* = \frac{v + v_*}{2} - \frac{|v - v_*|}{2}\sigma,$$

which are obtained in such a way that collision preserves momentum and kinetic energy, namely

$$v'_* + v' = v + v_*, \quad |v'_*|^2 + |v'|^2 = |v|^2 + |v_*|^2.$$

For monatomic gas, the collision cross section $B(v - v_*, \sigma)$ is a non-negative function which depends only on $|v - v_*|$ and $\cos\theta$ which is defined by the scalar product

$$\cos\theta = \frac{v - v_*}{|v - v_*|} \cdot \sigma.$$

Without loss of generality, we may assume that $B(v - v_*, \sigma)$ is supported on the set $\cos\theta \geq 0$, i.e. where $0 \leq \theta \leq \frac{\pi}{2}$. See for example [9], [29] for more explanations about the support of θ. For physical models, the collision cross section usually takes the form

$$B(v - v_*, \sigma) = \Phi(|v - v_*|)b(\cos\theta),$$

with a kinetic factor

$$\Phi(|v - v_*|) = |v - v_*|^\gamma, \ \gamma \in]-3, +\infty[.$$

The molecules are said to be Maxwellian when the parameter $\gamma = 0$.

Except for the hard sphere model, the function $b(\cos\theta)$ has a singularity at $\theta = 0$. For instance, in the important model case of the inverse-power potentials,

$$U(\rho) = \frac{1}{\rho^r}, \ \text{with } r > 1,$$

with ρ being the distance between two interacting particles in the physical 3-dimensional space $\mathbb{R}^3$,

$$b(\cos\theta)\sin\theta \sim K\theta^{-1-2s}, \quad s = \frac{1}{r} \text{ as } \theta \to 0^+.$$

The notation $a \sim b$ means that there exist positive constants $C_2 > C_1 > 0$, such that

$$C_1 a \leq b \leq C_2 a.$$