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A state-dependent queueing system with asymptotic logarithmic distribution

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A B S T R A C T

A Markovian single-server queueing model with Poisson arrivals and state-dependent service rates, characterized by a logarithmic steady-state distribution, is considered. The Laplace transforms of the transition probabilities and of the densities of the first-passage time to zero are explicitly evaluated. The performance measures are compared with those ones of the well-known $M/M/1$ queueing system. Finally, the effect of catastrophes is introduced in the model and the steady-state distribution, the asymptotic moments and the first-visit time density to zero state are determined.

1. Introduction

Queueing theory plays an important role in wide areas of science, technology and management. Applications of queueing can be seen in traffic modeling, business and industries, computer-communication, health sectors and medical sciences, etc. In the study of queueing systems the emphasis is often placed in obtaining steady-state performance measures, but in many applications it is necessary to know the behavior of the system not only in the asymptotic regime but also in the transient phase.

In birth–death queueing models the instantaneous arrival and departure rates depend on the number of customers in the system. A systematic study of the birth–death queue with varying arrival and service rates has been carried out by Abate, Conolly, Chan, Gupta and Srinivasa Rao, Hadidi, Kyriakidis, Natvig, Parthasarathy and Servaraju, Sharma, Sudhesh, Van Doorn. These authors give transient and stationary solutions for the queue length process, waiting time, busy period and output for special birth–death queues with adaptive demand and service mechanism (see [1,5,11–13,22,24,25,32–34,36,39,40,44]). The transient

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analysis of the state-dependent queueing systems often presents considerable difficulties and also numerical solutions are generally difficult to get. In some queueing systems, modeled as birth–death processes, it is assumed that new customers enter into the system according to a Poisson process with constant rate λ, so that the PASTA property holds; moreover, the servers may not work at a constant rate, but they adapt their behavior to the state of the system by speeding up to empty the queue or by slowing down when they are overworked.

The busy period and the first-passage time (FPT) to state 0 play a relevant role in a queueing system with state-dependent arrival and service rates (cf. [10,23,26]). The busy period for a single-server system is the time interval between any two successive idle periods. For a single-server queueing system, a busy period is equivalent to a FPT from state 1 to state 0. The analysis of the FPT distributions and their moments is helpful for the efficient planning of the system. In [26], Jouini and Dallery derive closed-form expressions for FPT moments of a general birth–death process; furthermore, they compute the moments of the busy period for some Markovian queues.

Recently there has been a rapid increase in the literature of stochastic models which are subject to catastrophes; some relevant results on this topic are given in [4,18,19,35,37]. In particular, birth–death models with catastrophes have been discussed in the context of population dynamics (see, for instance, [6,7,31,42]) and in queueing systems (see, for instance, [8,15,17,29,30,40]). Whenever a catastrophe occurs in a queueing system, all the customers are destroyed immediately, the server remains inactive momentarily and it is ready for service when a new arrival occurs. For instance, queueing models with disasters can be used to analyze system breakdowns due to a reset order or computer networks with virus infections. In [16] and [18] various functional relations are given to describe the birth–death process in the presence of catastrophes in terms of the birth–death process without catastrophes, characterized by the same birth and death rates. Furthermore, in [16] the problem of first-visit time (FVT) to state 0 and of the first occurrence of an effective catastrophe are discussed.

In this paper we investigate a single-server queueing system with Poisson arrivals (interarrival intervals of type M) and a special state-dependent service mechanism, assuming an infinite waiting-room and a first-come-first-served queueing discipline. In Section 2, under suitable assumptions about arrival and departure rates, we give some preliminary results on the steady-state distributions of birth–death processes related to the Sundt–Jewell class and to stochastic orderings. Subsequently, we focus on the study of the birth–death process with constant arrival rates $\lambda_n = \lambda$ and with state-dependent service rates μ_n, such that $\mu_1 = \mu$ and $\mu_n = \mu\, n/(n-1)$ for $n = 2, 3, \ldots$, being n the number of customers present in the system. For this model, in Section 3 the asymptotic analysis is carried out and the Stein operator is derived for the steady-state distribution. The Laplace transforms of the transition probabilities and of the first two moments are obtained. In Section 4, the Laplace transforms of the FPT pdf to 0 are determined; furthermore, the mean and the variance of the busy period are studied. In Section 5, we include the effect of total catastrophes in the model. Catastrophes occur with exponential rate ξ and reduce the number of customers instantaneously to 0. For the model with catastrophes, the steady-state distribution and the first two asymptotic moments are determined. The Laplace transform of FVT pdf to 0 is given. Finally, the busy period of the process with catastrophes is analyzed and the mean and the variance are provided. The obtained results for the model without (with) catastrophes are compared with the corresponding results for the $M/M/1$ queue.

2. Preliminaries

Let $\{N(t), t \geq 0\}$ be a continuous-time birth–death process on the state-space $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$, with birth rates $\lambda_n > 0$ ($n \in \mathbb{N}_0$) and death rates $\mu_0 = 0$, $\mu_n > 0$ ($n \in \mathbb{N}$). The transition probabilities $p_{j,n}(t) := P\{N(t) = n | N(0) = j\}$ ($n, j \in \mathbb{N}_0$) satisfy the forward Kolmogorov equations: