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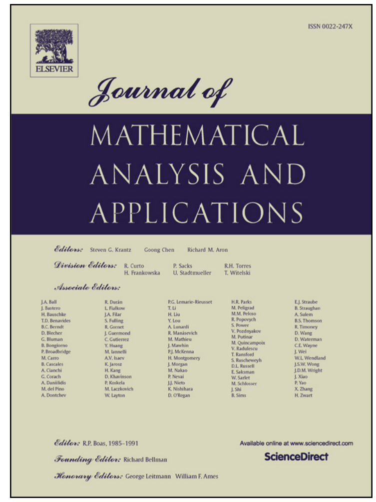

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Multiplicity of solutions for singular quasilinear Schrödinger equations with critical exponents*

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Abstract: By variational methods, we study the existence of multiplicity of solutions for singular quasilinear Schrödinger equations.

Key works: Singular quasilinear Schrödinger equation; genus; critical exponents

AMS Subject Classification(2010): 35B33; 35J20; 35J60; 35Q55

1 Introduction

In this paper we consider the existence of multiplicity of solutions for the following singular quasilinear Schrödinger equations:

$$-\Delta u - \frac{\alpha}{2}\Delta(|u|^{\alpha})|u|^{\alpha-2}u = \lambda V(x)|u|^{p-2}u + \beta K(x)|u|^{2^*-2}u, \quad x \in \mathbb{R}^N, \tag{1.1}$$

where $0 < \alpha < 1$, $1 < p < 2^* = \frac{2N}{N-2}$, $N \geq 3$, λ, $\beta \in \mathbb{R}$, $K(x) \in C(\mathbb{R}^N) \cap L^{\infty}(\mathbb{R}^N)$, $0 \leq V(x) \in C(\mathbb{R}^N) \cap L^q(\mathbb{R}^N)$ with $q = \frac{2^*}{2^*-p}$. Equation of type (1.1) comes from the following general problems:

$$-\Delta u + V(x)u - \frac{\alpha}{2}\Delta(|u|^{\alpha})|u|^{\alpha-2}u = h(u), \quad x \in \mathbb{R}^N, \tag{1.2}$$

where $V(x) : \mathbb{R}^N \to \mathbb{R}$ is a given potential and h is a real function. For $\alpha = 2$, there are an enormous number of investigations on (1.2), we refer to [10, 11, 12, 14, 15, 17, 18] and references therein. For general $\alpha > 1$, Liu and Wang [9] established the existence of ground states of (1.2) with $h(t) = \lambda|t|^{p-2}t$, $2 < p < \alpha 2^*$ by minimization arguments. Later, the existence of G-invariant positive solution for $\alpha \geq \frac{3}{2}$ and the uniqueness of ground state solution for $\alpha > 1$ were studied by Adachi et al. in [4] and [5], respectively. In [19], Shen and Wang studied (1.2) with $\alpha \geq \frac{3}{2}$ and $h(t) = |t|^{\alpha 2^*-2}t + f(t)$. Under some suitable conditions posed on $V(x)$ and

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