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Bounded point evaluations for rationally multicyclic subnormal operators

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ABSTRACT

Let S be a pure bounded rationally multicyclic subnormal operator on a separable complex Hilbert space $\mathcal{H}$ and let M_z be the minimal normal extension on a separable complex Hilbert space $\mathcal{K}$ containing $\mathcal{H}$. Let $bpe(S)$ be the set of bounded point evaluations and let $abpe(S)$ be the set of analytic bounded point evaluations. We show $abpe(S) = bpe(S) \cap Int(\sigma(S))$. The result affirmatively answers a question asked by J. B. Conway concerning the equality of the interior of $bpe(S)$ and $abpe(S)$ for a rationally multicyclic subnormal operator S. As a result, if $\lambda_0 \in Int(\sigma(S))$ and $dim(ker(S - \lambda_0)^*) = N$, where N is the minimal number of cyclic vectors for S, then the range of $S - \lambda_0$ is closed, hence, $\lambda_0 \in \sigma(S) \setminus \sigma_e(S)$.

1. Introduction

Let $\mathcal{H}$ be a separable complex Hilbert space and let $\mathcal{L}(\mathcal{H})$ be the space of bounded linear operators on $\mathcal{H}$. An operator $S \in \mathcal{L}(\mathcal{H})$ is subnormal if there exist a separable complex Hilbert space $\mathcal{K}$ containing $\mathcal{H}$ and a normal operator $M_z \in \mathcal{L}(\mathcal{K})$ such that $M_z\mathcal{H} \subset \mathcal{H}$ and $S = M_z|_{\mathcal{H}}$. By the spectral theorem of normal operators, we assume that

$$\mathcal{K} = \oplus_{i=1}^{m} L^2(\mu_i) \tag{1.1}$$

where $\mu_1 >> \mu_2 >> ... >> \mu_m$ (m may be ∞) are compactly supported finite positive measures on the complex plane $\mathbb{C}$, and M_z is multiplication by z on $\mathcal{K}$. For $H = (h_1, ..., h_m) \in \mathcal{K}$ and $G = (g_1, ..., g_m) \in \mathcal{K}$, we define

$$\langle H(z), G(z)\rangle = \sum_{i=1}^{m} h_i(z)\overline{g_i(z)}\frac{d\mu_i}{d\mu_1},\ |H(z)|^2 = \langle H(z), H(z)\rangle\,. \tag{1.2}$$

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The inner product of H and G in $\mathcal{K}$ is defined by

$$(H, G) = \int \langle H(z), G(z) \rangle \, d\mu_1(z). \tag{1.3}$$

M_z is the minimal normal extension if

$$\mathcal{K} = clos\left(span(M_z^{*k}x : \; x \in \mathcal{H}, \; k \geq 0)\right). \tag{1.4}$$

We will always assume that M_z is the minimal normal extension of S and $\mathcal{K}$ satisfies (1.1) and (1.4). For details about the functional model above and basic knowledge of subnormal operators, the reader shall consult Chapter II of the book Conway [10].

For $T \in \mathcal{L}(\mathcal{H})$, we denote by $\sigma(T)$ the spectrum of T, $\sigma_e(T)$ the essential spectrum of T, T^* its adjoint, $ker(T)$ its kernel, and $Ran(T)$ its range. For a subset $A \subset \mathbb{C}$, we set $Int(A)$ for its interior, $\bar{A}$ or $clos(A)$ for its closure, A^c for its complement, and χ_A for its characteristic function. Let $\delta_{ij} = 1$ when $i = j$ and $\delta_{ij} = 0$ when $i \neq j$. For $\lambda \in \mathbb{C}$ and $\delta > 0$, we set $B(\lambda, \delta) = \{z : |z - \lambda| < \delta\}$ and $\mathbb{D} = B(0, 1)$. Let $\mathcal{P}$ denote the set of polynomials in the complex variable z. For a compact subset $K \subset \mathbb{C}$, let $Rat(K)$ be the set of all rational functions with poles off K.

A subnormal operator S on $\mathcal{H}$ is pure if for every non-zero invariant subspace I of S ($SI \subset I$), the operator $S|_I$ is not normal. For $F_1, F_2, ..., F_N \in \mathcal{H}$, let

$$R^2(S|F_1, F_2, ..., F_N) = clos\{r_1(S)F_1 + r_2(S)F_2 + ... + r_N(S)F_N\} \tag{1.5}$$

in $\mathcal{H}$, where $r_1, r_2, ..., r_N \in Rat(\sigma(S))$ and let

$$P^2(S|F_1, F_2, ..., F_N) = clos\{p_1(S)F_1 + p_2(S)F_2 + ... + p_N(S)F_N\} \tag{1.6}$$

in $\mathcal{H}$, where $p_1, p_2, ..., p_N \in \mathcal{P}$. A subnormal operator S on $\mathcal{H}$ is rationally multicyclic (N-cyclic) if there are N vectors $F_1, F_2, ..., F_N \in \mathcal{H}$ such that

$$\mathcal{H} = R^2(S|F_1, F_2, ..., F_N)$$

and for any $G_1, ..., G_{N-1} \in \mathcal{H}$,

$$\mathcal{H} \neq R^2(S|G_1, G_2, ..., G_{N-1}).$$

S is multicyclic (N-cyclic) if

$$\mathcal{H} = P^2(S|F_1, F_2, ..., F_N)$$

and for any $G_1, ..., G_{N-1} \in \mathcal{H}$,

$$\mathcal{H} \neq P^2(S|G_1, G_2, ..., G_{N-1}).$$

In this case, $m \leq N$ where m is as in (1.1).

Let μ be a compactly supported finite positive measure on the complex plane $\mathbb{C}$ and let $spt(\mu)$ denote the support of μ. For a compact subset K with $spt(\mu) \subset K$, let $R^2(K, \mu)$ be the closure of $Rat(K)$ in $L^2(\mu)$. Let $P^2(\mu)$ denote the closure of $\mathcal{P}$ in $L^2(\mu)$.

If S is rationally cyclic, then S is unitarily equivalent to multiplication by z on $R^2(\sigma(S), \mu_1)$, where $m = 1$ and $F_1 = 1$. We may write $R^2(S|F_1) = R^2(\sigma(S), \mu_1)$. If S is cyclic, then S is unitarily equivalent to multiplication by z on $P^2(\mu_1)$. We may write $P^2(S|F_1) = P^2(\mu_1)$.