J. Math. Anal. Appl. ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Blow-up criteria for three-dimensional compressible radiation hydrodynamics equations with vacuum

Yachun Li [a], Shuai Xi [b,*]

[a] School of Mathematical Sciences, MOE-LSC, and SHL-MAC, Shanghai Jiao Tong University, Shanghai 200240, PR China
[b] School of Mathematical Sciences, Shanghai Jiao Tong University, Shanghai 200240, PR China

ARTICLE INFO

Article history:
Received 29 August 2016
Available online xxxx
Submitted by J. Guermond

Keywords:
Navier–Stokes–Boltzmann
Strong solutions
Vacuum
Blow-up criterion
Radiation

ABSTRACT

We consider the blow-up criteria for the Cauchy problems of three-dimensional compressible radiation fluids with vacuum. It is shown to own the same BKM-type criterion as the compressible Navier–Stokes equations [14], while the $L^{\tilde{p}}$ ($\tilde{p} \in [2, 3]$) norm of the density gradient should be involved for the Serrin-type criterion.

© 2017 Published by Elsevier Inc.

1. Introduction

This paper is concerned with the blow-up criteria to the strong solution with vacuum for the three-dimensional compressible isentropic radiation hydrodynamics (RHD) equations. The system is used in various astrophysical contexts [18] and in high-temperature plasma physics [17]. The couplings between fluid field and radiation field involve momentum source and energy source depending strongly on the specific radiation intensity derived by the so called radiative transfer integro-differential equation [28]. In particular, if the matter is in the local thermodynamical equilibrium (LTE), the system can be governed by the following Navier–Stokes–Boltzmann equations:

$$\begin{cases} \frac{1}{c} I_t + \Omega \cdot \nabla I = A_r, \quad (v, \Omega, t, x) \in \mathbb{R}^+ \times S^2 \times \mathbb{R}^+ \times \mathbb{R}^3, \\ \rho_t + \operatorname{div}(\rho u) = 0, \\ \left(\rho u + \frac{1}{c^2} F_r\right)_t + \operatorname{div}(\rho u \otimes u + P_r) + \nabla p_m = \operatorname{div}\mathbb{T}, \end{cases} \tag{1.1}$$

* Corresponding author.
E-mail addresses: ycli@sjtu.edu.cn (Y. Li), shuai_xi@sina.com (S. Xi).

https://doi.org/10.1016/j.jmaa.2017.10.001
0022-247X/© 2017 Published by Elsevier Inc.

Please cite this article in press as: Y. Li, S. Xi, Blow-up criteria for three-dimensional compressible radiation hydrodynamics equations with vacuum, J. Math. Anal. Appl. (2018), https://doi.org/10.1016/j.jmaa.2017.10.001

where $\rho(t,x)$, $u(t,x)=(u_1,u_2,u_3)$ and $I(v,\Omega,t,x)$ denote the density, velocity field and specific radiation intensity, respectively.

In this system, S^2 is the unit sphere in $\mathbb{R}^3$, v is the frequency of photon and Ω is the travel direction of photon. The associated material pressure p_m is given by the state equation

$$p_m = A\rho^\gamma \tag{1.2}$$

for some positive constant A and adiabatic index $\gamma > 1$. Meanwhile, the stress tensor $\mathbb{T}$ equals

$$\mathbb{T} = \mu\left(\nabla u + (\nabla u)^\top\right) + \lambda\,(\mathrm{div}u)\,\mathbb{I}_3, \tag{1.3}$$

where $\mathbb{I}_3$ is the 3×3 unit matrix, $\mu > 0$ and $\lambda + \frac{2}{3}\mu \geq 0$ are the shear viscosity coefficient and bulk viscosity coefficient, respectively. These ensure the ellipticity of the Lamé operator.

As to the radiation part, the radiation flux F_r and the radiation pressure tensor P_r are defined by

$$F_r = \int_0^\infty\int_{S^2} I(v,\Omega,t,x)\Omega\mathrm{d}\Omega\mathrm{d}v,\ \ P_r = \frac{1}{c}\int_0^\infty\int_{S^2} I(v,\Omega,t,x)\Omega\otimes\Omega\mathrm{d}\Omega\mathrm{d}v,$$

and the collision term A_r in radiation transfer equation can be expressed as

$$A_r = S - \sigma_a I + \int_0^\infty\int_{S^2}\left(\frac{v}{v'}\sigma_s I' - \sigma_s' I\right)\mathrm{d}\Omega'\mathrm{d}v',$$

in which $I' = I(v',\Omega',t,x)$; $S = S(v,\Omega,t,x) \geq 0$ denotes the rate of energy emission due to spontaneous process; $\sigma_a = \sigma_a(v,\Omega,t,x,\rho) \geq 0$ is the absorption coefficient that may also depend on the mass density ρ; The differential scattering coefficient σ_s has two different state transitions:

$$\sigma_s \equiv \sigma_s(v' \to v, \Omega'\cdot\Omega, \rho) = O(\rho),\ \ \sigma_s' \equiv \sigma_s(v \to v', \Omega\cdot\Omega', \rho) = O(\rho).$$

Studying the radiation hydrodynamics equations is challenging because of its complexity and mathematical difficulty. For Navier–Stokes–Boltzmann equations, under some physical assumptions with the mass density away from vacuum, the local classical solution of the Cauchy problems was studied by Chen–Wang [6]. Ducomet and Nečasová [7,8] established the global weak solutions and the large time behavior in 1-D space. The local existence of strong solutions with vacuum was first established by Li–Zhu [23] when the initial data are arbitrarily large. They [22] also considered the formation of singularities to classical solutions when the initial mass density is compactly supported. For the inviscid radiation hydrodynamics equations (i.e., Euler–Boltzmann equations), we refer to [15–17,24].

As it was shown in [22], if we assume $\sigma_s = 0$, from the induced process and LTE assumption, the Navier–Stokes–Boltzmann system (1.1) can be rewritten into

$$\begin{cases} \frac{1}{c}I_t + \Omega\cdot\nabla I = -K_a\left(I - \bar{B}(v)\right), & (v,\Omega,t,x)\in\mathbb{R}^+\times S^2\times\mathbb{R}^+\times\mathbb{R}^3, \\ \rho_t + \mathrm{div}(\rho u) = 0, & \\ (\rho u)_t + \mathrm{div}(\rho u\otimes u) + \nabla p_m = \frac{1}{c}\int_0^\infty\int_{S^2} K_a\left(I - \bar{B}(v)\right)\Omega\mathrm{d}\Omega\mathrm{d}v + \mathrm{div}\mathbb{T}, & \end{cases} \tag{1.4}$$

Please cite this article in press as: Y. Li, S. Xi, Blow-up criteria for three-dimensional compressible radiation hydrodynamics equations with vacuum, J. Math. Anal. Appl. (2018), https://doi.org/10.1016/j.jmaa.2017.10.001

Download English Version:

https://daneshyari.com/en/article/8900278

Download Persian Version:

https://daneshyari.com/article/8900278

Daneshyari.com