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The hybrid-penalized method for the Timoshenko's beam

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A B S T R A C T

In this paper we introduce the hybrid-penalized method, to show the global existence of at least one solution to the Signorini's problem for a Timoshenko Beam. The main advantage of this method is that we can improve all result about asymptotic behaviour to contact problem to any boundary conditions, which is not possible with multiplicative techniques. Moreover this method also allows us to show polynomial stability to partial dissipative models. Finally, we performed numerical experiments to highlight our theoretical results.

1. Introduction

We consider the contact problem for the Timoshenko beam configured over the interval $[0, L] \subset \mathbb{R}$ see [15, 24]

$$\rho A \varphi_{tt} = [S(x,t)]_x + F_1(x,t),$$
$$\rho I \psi_{tt} = [M(x,t)]_x - S(x,t) + F_2(x,t),$$

where φ, ψ denote the transverse displacement and the rotation angle of a filament, respectively. The functions ρ, A and I stand for the density of the body, area of the cross-section and the inertial model, respectively. By S and M we denote the shear stress and bending moment, respectively. The functions F_1 and F_2 represent the body force and moment acting outside the body. The constitutive law we use in this paper is given by

$$S = \kappa GA(\varphi_x + \psi), \qquad M = EI\psi_x,$$

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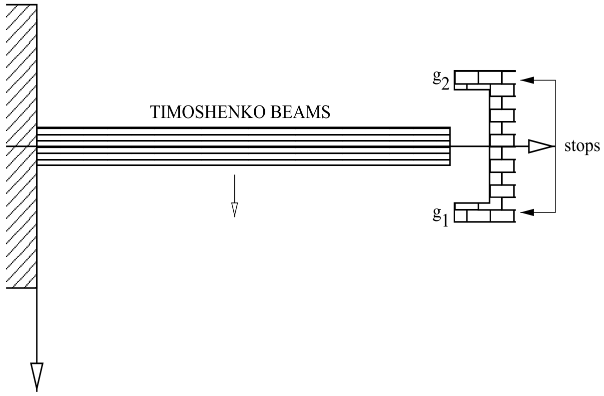

Fig. 1. Beam subject to a constraint at the free end L.

where κ is correction factor, E is the Young's modulus, and G is shear modulus. In the sequel, we consider all the parameters as positive constants and assume that $F_1(x,t) = f_1(x,t) - m\varphi_t$ and $F_2(x,t) = f_2(x,t) - d\psi_t$, where $m\varphi_t$ and $d\psi_t$ models a frictional mechanism effective over the whole beam.

Denoting as

$$\rho_1 = \rho A, \quad \rho_2 = \rho I, \quad k = \kappa GA, \quad b = EI,$$

the Timoshenko's model can be written as

$$\begin{aligned} &\rho_1\varphi_{tt} - k(\varphi_x + \psi)_x + m\varphi_t = f_1(x,t), \\ &\rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x + \psi) + d\psi_t = f_2(x,t). \end{aligned} \tag{1.1}$$

The boundary conditions we consider are given by

$$\varphi(0,t) = \psi(0,t) = \psi(L,t) = 0, \quad \forall t \in \ (0,T). \tag{1.2}$$

On the other hand, in the $x = L$-end we consider two obstacles that can produce contact in the transverse oscillations of the beam.

In Fig. 1 g_1 and g_2 are the gaps of the obstacle, therefore we have that

$$g_1 \le \varphi(L,t) \le g_2, \quad 0 \le t \le T. \tag{1.3}$$

In the contact problem we have the following situations

$$\begin{aligned} &S(L,t) > 0 \Rightarrow \varphi(L,t) = g_1, \\ &S(L,t) = 0 \Rightarrow g_1 < \varphi(L,t) < g_2, \\ &S(L,t) < 0 \Rightarrow \varphi(L,t) = g_2. \end{aligned} \tag{1.4}$$

Finally, we consider the initial conditions

$$\varphi(x,0) = \varphi_0(x), \quad \varphi_t(x,0) = \varphi_1(x), \ \forall x \in (0,L), \tag{1.5}$$

$$\psi(x,0) = \psi_0(x), \quad \psi_t(x,0) = \psi_1(x), \ \forall x \in (0,L). \tag{1.6}$$

Concerning the existence of solutions to Signorini's problem we have the pioneer work of Kim [12,13] who showed the global existence of at least one solution (until now the uniqueness of this problem is an

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