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Global attractor of the quasi-linear wave equation with strong damping ☆

Zhijian Yang *, Zhiming Liu

School of Mathematics and Statistics, Zhengzhou University, No. 100, Science Road, Zhengzhou 450001, China

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ABSTRACT

The paper investigates the existence of global attractor of the quasi-linear wave equation with strong damping $u_{tt} - \gamma\Delta u_t - \Delta u + f(u) = \nabla \cdot \phi'(\nabla u) + g$. It proves that the energy solution of the equation is stable and the related dynamical system possesses a (strong) global attractor in natural energy space (rather than a weak one as known before). These results improve the recent ones in [13].

1. Introduction

In this paper, we are concerned with the existence of global attractor of the quasi-linear wave equation with strong damping

$$u_{tt} - \gamma\Delta u_t - \Delta u + f(u) = \nabla \cdot \phi'(\nabla u) + g \quad \text{in} \quad \Omega \times \mathbb{R}^+, \tag{1.1}$$

$$u|_{\partial\Omega} = 0, \quad (u(0),\ u_t(0)) = (u_0, u_1), \tag{1.2}$$

where Ω is a bounded domain in $\mathbb{R}^3$ with the smooth boundary $\partial\Omega, \gamma$ is a positive constant, $\phi(\eta)$ and $f(s)$ are nonlinear functions specified later, and g is an external force term.

Eq. (1.1) is a class of essential quasi-linear wave models: one typical example is $\phi(\eta) = |\eta|^{p+1}$, that is, $\phi'(\eta) = (p+1)|\eta|^{p-1}\eta$, $\eta \in \mathbb{R}^3$ (p-Laplacian) (see e.g. [1,2,7,10,12,17,21]); another one is $\phi(\eta) = \sqrt{1+|\eta|^2}$, that is, $\phi'(\eta) = \frac{\eta}{\sqrt{1+|\eta|^2}}$ (see e.g. [11,14–16]). These models govern the motion of a fixed membrane with strong viscosity.

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* Corresponding author.
E-mail addresses: yzjzzut@tom.com (Z. Yang), lzming126@163.com (Z. Liu).

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There have been extensive concerns on the well-posedness and asymptotic behavior of solutions to the equations related to type (1.1) (cf. [1,2,10,12,14–16] and references therein). Global attractor is a basic concept to study the longtime dynamics of nonlinear evolution equations with various dissipations. When $\Omega = (0,1)$, Chen et al. [7] established in phase space $H^2(\Omega) \times H_0^1(\Omega)$ the existence of global attractor of the equation

$$u_{tt} - \partial_x \sigma(u_x) - u_{xxt} + f(u) = g. \tag{1.3}$$

But in multi-dimensional case, the research on the global attractor of Eq. (1.1) is much more complicated. Recently, Kalantarov and Zelik [13] investigated the well-posedness and the existence of global and exponential attractor as well as their additional regularity for problem (1.1)–(1.2) in the case where the growth exponent of the non-linearity ϕ is less than 6 and f may have arbitrary polynomial growth. Their results cover the above mentioned examples.

But the global attractor $\mathcal{A}$ obtained in [13] is a (weak) $(\mathcal{E}, \mathcal{E}_{-1})$ one (see below for the definitions of $\mathcal{E}$ and $\mathcal{E}_{-1}$). Although $\mathcal{A}$ has finite fractal dimension in $\mathcal{E}$ and is bounded in a more regular space $\mathcal{E}_1$ (see [13]), the attractiveness of $\mathcal{A}$ is in the topology of weaker space $\mathcal{E}_{-1}$ rather than in that of natural energy space $\mathcal{E}$. The purpose of the present paper is to show that actually this attractor $\mathcal{A}$ is a "strong" global attractor, that is, $\mathcal{A}$ attracts the bounded sets of $\mathcal{E}$ in the topology of $\mathcal{E}$.

Recently, taking $\phi(\eta) = \sqrt{1+|\eta|^2}$ and replacing the strong damping $-\Delta u_t$ in Eq. (1.1) by structural damping $(-\Delta)^\alpha u_t$, with $1/2 < \alpha < 1$, we have studied the well-posedness and longtime dynamics of the quasi-linear wave equations

$$u_{tt} - \Delta u + (-\Delta)^\alpha u_t - \nabla \cdot \left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}} \right) + f(u) = g. \tag{1.4}$$

It is showed that Eq. (1.4) is like parabolic because of the effectiveness of structural damping though it is a wave equation, that is, its weak energy solution is of global smooth property (rather partial one as known before) for $t > 0$, which leads to that there exists Sobolev compact embedding from the regularized phase space to the original energy space, so the related solution semigroup has a global and an exponential attractor in natural energy space (see [22]).

Unfortunately, the techniques used in [22] is useless here because Eq. (1.1) is only partially like parabolic (see Theorem 2.4). The essential difficulties to establish the existence of (strong) global attractor $\mathcal{A}$ are that one can not get the (strong) attractiveness of $\mathcal{A}$ in the spaces $W_0^{1,p+1}$ and L^{q+2} because the weak energy solution possesses only partial regularity for $t > 0$, which leads to that there is no Sobolev (compact) embedding into the above-mentioned spaces.

Recently, J. Ball [3,4] proposed the concept of generalized semiflows and used it (see [4]) to establish the existence of global attractor for the damped semilinear wave equation with supercritical nonlinearity

$$u_{tt} - \Delta u + \gamma u_t + f(u) = g. \tag{1.5}$$

He proposed a new technique to study the existence of global attractor though his proof is based on an unproved assumption that every weak solution satisfies the energy identity.

The motivation of the present paper comes from [4,13]. On the basis of the results in [13] (for the authors' convenience, we use some same functional signs as in [13]), we further find some new properties of the weak energy solutions and use them to establish the energy identity (which holds only for the Galerkin approximations in [13] rather than for the energy solution here) and the continuity of related solution semigroup $S(t)$ in phase space $\mathcal{E}$ (see Theorem 3.3), then prove the existence of the (strong) global attractor by virtue of J. Ball's technique (cf. [4]) (see Theorem 3.4). These results improve the recent ones in [13].

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