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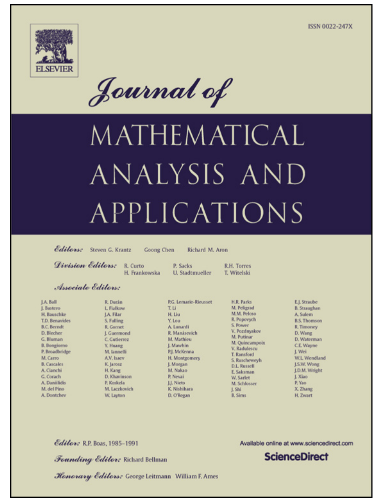

Blow-up analyses in parabolic equations with anisotropic nonstandard damping source

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Blow-up analyses in parabolic equations with anisotropic nonstandard damping source

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Abstract. In this paper, we consider the nonlinear parabolic problems with anisotropic nonstandard growth conditions and damping terms. After obtaining well-posedness of solutions, we give blow-up criteria of solutions through constructing different control functions and generalizing eigenfunction method, respectively. We also classify global solutions in all scope of the variable exponents. Moreover, the sharp blow-up rates, blow-up time and blow-up set are determined, which seem to be rarely studied for parabolic problems with anisotropic nonstandard damping sources. It is interesting that the asymptotic estimates of blow-up solutions rely not only on maxima and minima of the anisotropic exponents, but also on the geometry properties of the spatial domain and the scope of variable coefficients.
MSC (2010): 35K55, 35B40, 35K15, 35B33.
Keywords: anisotropic variable exponent; blow-up rate; blow-up set; blow-up time; damping source.

1 Introduction

Nonlinear parabolic problems with nonstandard growth conditions (for example, with anisotropic or isotropic variable exponents) come from several branches of applied mathematics and physics, such as, flows of electro-rheological or thermo-rheological fluids, and the processing of digital images (see [1]–[5], [8, 9, 13, 26, 37]). In the present paper, we consider the following parabolic problems with anisotropic nonstandard growth conditions:

$$\begin{cases} u_t = \Delta u + b(x,t)u^{p(x,t)} \displaystyle\int_\Omega u^{q(x,t)}\mathrm{d}x - k(x,t)u^{m(x,t)}, & (x,t) \in Q_T := \Omega \times (0,T), \\ u(x,t) = 0, & (x,t) \in \Gamma_T := \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) \geq 0, & x \in \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset R^N$ is a simple-connected and bounded domain with Lipschitz boundary $\partial\Omega$; Let $T(\leq +\infty)$ be the maximal existence time of (1.1); $b(x,t)$, $k(x,t)$, $p(x,t)$, $q(x,t)$ and $m(x,t)$ are Hölder-continuous in Q_T with the notations, e.g., $p^- := \inf_{Q_T} p(\cdot,\cdot)$ and $p^+ := \sup_{Q_T} p(\cdot,\cdot)$, satisfying that

$$0 < p^- \leq p(x,t) \leq p^+ < +\infty, \quad 0 < q^- \leq q(x,t) \leq q^+ < +\infty, \quad 0 < m^- \leq m(x,t) \leq m^+ < +\infty,$$
$$0 < b^- \leq b(x,t) \leq b^+ < +\infty, \qquad 0 < k^- \leq k(x,t) \leq k^+ < +\infty.$$

The solution $u(x,t)$ of (1.1) may describe some kind of entropy per volume for some material, which can be dissipative by the nonlinear damping effect in the non-homogeneous and anisotropic medium, and be accumulated by positive local and spatial-homogeneous nonlinearities, respectively.

The diffusion problems, such as (1.1) with constant exponents, describe the models in population dynamics, nuclear technology and biological sciences. Wang and Wang in [41] studied the problems:

$$u_t = \Delta u + \int_\Omega u^q \mathrm{d}x - ku^m, \quad (x,t) \in Q_T, \tag{1.2}$$

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