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Lipschitz equivalence of self-similar sets with two-state neighbor automaton ☆

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ABSTRACT

The study of Lipschitz equivalence of fractals is a very active topic in recent years, but there are very few results on fractals which is not totally disconnected. In this paper, using finite state automata and the angle separation property, we prove that for a class of self-similar sets with two-state neighbor automaton, two elements are Lipschitz equivalent if and only if they have the same Hausdorff dimension.
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1. Introduction

Two metric spaces (X, d_X) and (Y, d_Y) are said to be *Lipschitz equivalent*, and denote by $X \sim Y$, if there is a bijection $f: X \to Y$ and a constant $C > 0$ such that

$$C^{-1} d_X(x_1, x_2) \le d_Y\big(f(x_1), f(x_2)\big) \le C d_X(x_1, x_2), \quad \forall \ x_1, x_2 \in X.$$

We call f a *bi-Lipschitz mapping* and C a *Lipschitz constant.*

The study of Lipschitz equivalence of fractal sets was initialled by Cooper and Pignartaro [2], Falconer and Marsh [6], David and Semmes [3], and it becomes a very active topic recently. The studies focus on two directions. The first direction focuses on dust-like self-similar sets, see [12,24,7,16], where the main concern is to construct Lipschitz invariant. The other direction focuses on a special class of fractals, called fractal cubes, where the main difficulty is to construct bi-Lipschitz map between fractal squares ([14,22,23,9,10,19]). A survey is provided by [13].

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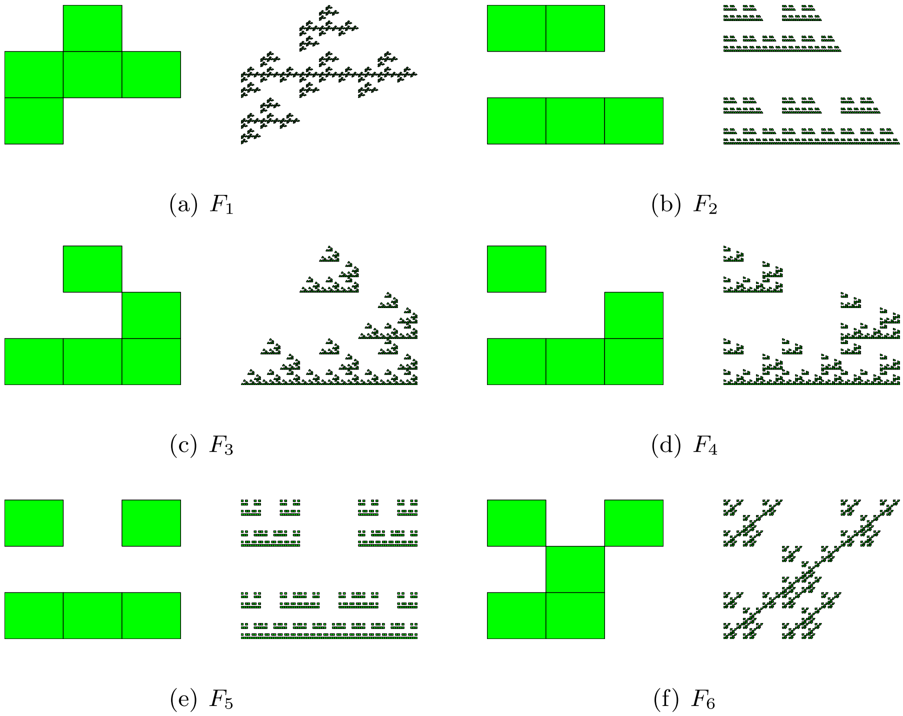

Fig. 1. Neither connected nor totally disconnected fractal squares with $n = 3$, $\#\mathcal{D} = 5$.

A family of contractions $\{\varphi_j\}_{j=1}^m$ on $\mathbb{R}^d$ is called an *iterated function system* (IFS). The unique nonempty compact set K satisfying $K = \bigcup_{j=1}^m \varphi_j(K)$ is called the attractor of the IFS. If the contractions are all similitudes, then the attractor is called a *self-similar set* defined by the IFS (cf. [5]).

For $n \geq 2$, let $\mathcal{D} = \{d_1, \cdots, d_m\} \subseteq \mathbb{R}^d$ and call it a *digit set*. Let $\{\varphi_j\}_{j=1}^m$ be the IFS on $\mathbb{R}^d$ given by $\varphi_j(x) = \frac{1}{n}(x + d_j)$, and let $K = K(n, \mathcal{D})$ be its attractor. Then

$$K = (K + \mathcal{D})/n. \tag{1.1}$$

When the digit set $\mathcal{D} \subset \{0, 1, \ldots, n-1\}^d$, K is called a d-dimensional *fractal cube* in [22] and [8].

David and Semmes [3] conjectured the fractal intervals $K(5, \{0, 2, 4\})$ and $K(5, \{0, 3, 4\})$ are not Lipschitz equivalent. Rao, Ruan and Xi [14] showed that the conjecture is false. The following theorem is proved by [14] in the 1-dimensional case, and Xi and Xiong [22] settled the general case.

Theorem 1.1. *([14] [22]) Let $K(n, \mathcal{D}_1)$ and $K(n, \mathcal{D}_2)$ be two totally disconnected fractal cubes, then $K(n, \mathcal{D}_1) \sim K(n, \mathcal{D}_2)$ if and only if $\#\mathcal{D}_1 = \#\mathcal{D}_2$, where $\#A$ denotes the cardinality of a set A.*

The study of Lipschitz equivalence of non-totally disconnected fractals is a difficult problem, and it is started only several years ago. There are several papers attempt to classify the fractal squares with contraction ratio 1/3 by Lipschitz equivalence, see Wen et al. [20] (the case $\#\mathcal{D} = 4$), Rao et al. [15] (the case $\#\mathcal{D} = 6$), Ruan and Wang [18] (the case $\#\mathcal{D} = 7, 8$). Luo and Liu [11] classifies connected fractal squares in the case $\#\mathcal{D} = 5$, and showed that in this case, the fractal squares which are connected but contains non-trivial connected components belong to 6 different classes with respect to linear transformations. These 6 classes are depicted in Fig. 1. Using the theory of finite state automaton, Rao and Zhu [17] showed that $F_1 \sim F_3$. One motivation of the present paper is to show that $F_1 \sim F_2$.

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